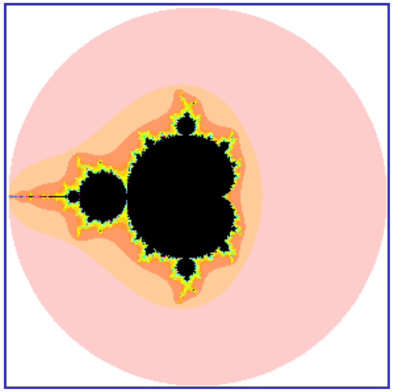
Introduction to Fractal Geometry and its Applications

College Visit

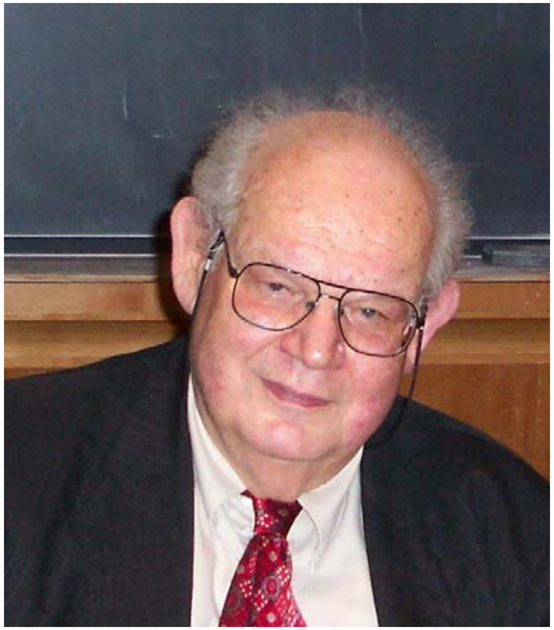
Final (Three Hour) Workshop

Middle School Slides

Fractals



- Fractals can be described as
  - Broken
  - Fragmented
  - Irregular
- Concept created by Benoit Mandelbrot to describe nature and measure roughness.



Picture of Benoit B. Mandelbrot was taken at his lecture at Worcester Polytechnic Institute, November 2006 and the picture of the Mandelbrot set is from: The fractal geometry Web site, http://classes.yale.edu/fractals/ of Michael Frame, Benoit Mandelbrot and Nial Neger. Courtesy of Michael Frame.

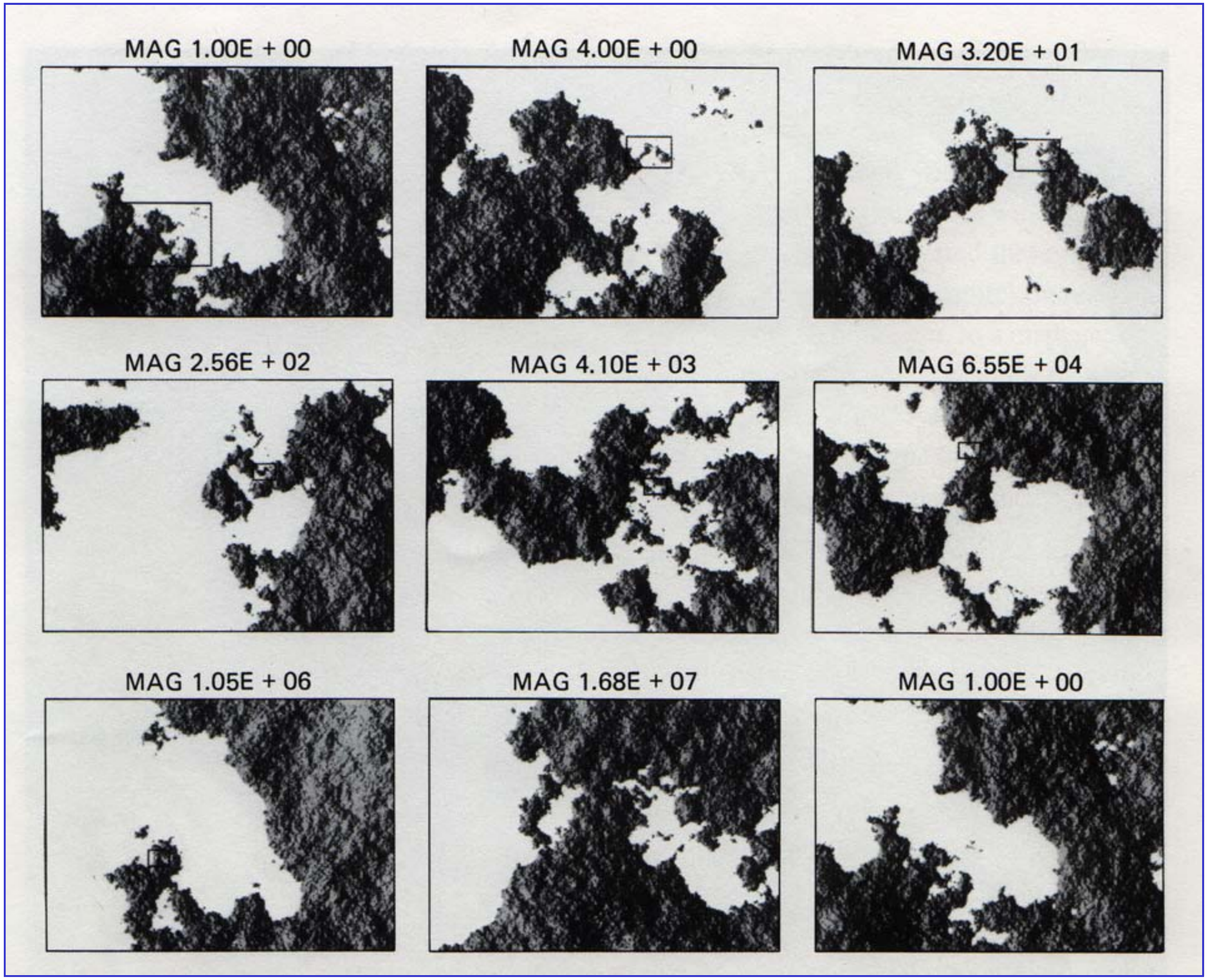
Historical Perspective

"I coined *fractal* from the Latin adjective *fractus.* The corresponding Latin verb *frangere* means 'to break:' to create irregular fragments. " *Benoit B. Mandelbrot*

Reference: Mandelbrot, Benoit B. *The Fractal Geometry of Nature*. San Francisco: W. H. Freeman and Company, 1977. 4.

Reference: Michael Frame, *Natural and Manufactured Fractals*, http://classes.yale.edu/Fractals//welcome.html.
Courtesy of Michael Frame.

Geometry of Nature



Regardless of the scale, the actual coastlines appear to have the same amount of jaggedness.

Courtesy of Prof. Dr. Heinz-Otto Peitgen for the coastline images: Barnsley, Devaney, Mandelbrot, Peitgen, Saupe and Voss. *The Science of Fractal Images*. *New York:* Springer-Verlag, 1988. 23.

Reference: Janson, H. W. *History of Art*. New York: Prentice-hall, Inc. and Harry N. Abrams, Inc.; 1964. 504.

Paul Cézanne, French, 1839-1906
Mont Sainte-Victoire Seen from the Bibémus Quarry, 1897
Oil on canvas
25-1/8 x 31-1/2 in. (65.1 x 80 cm.)
Courtesy of: The Baltimore Museum of Art: The Cone Collection, formed by Dr. Claribel Cone and Miss Etta Cone of Baltimore, Maryland
BMA 1950.196

Cezanne's statement about painting: "Everything in Nature can be viewed in terms of cones, cylinders and spheres." Can you find a cone shape? A cylinder?

Reference: Michael Frame, *Natural and Manufactured Fractals*, http://classes.yale.edu/fractals/

1960 – 1990s Mandelbrot at IBM Research

NSF Geometry Super Computer Project

“Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in straight lines.”

Mandelbrot

Reference: Mandelbrot, Benoit B. *The Fractal Geometry of Nature*. San Francisco: W. H. Freeman and Company, 1977. 1.

Today

Chaos and Dynamical Systems are current fields of research.

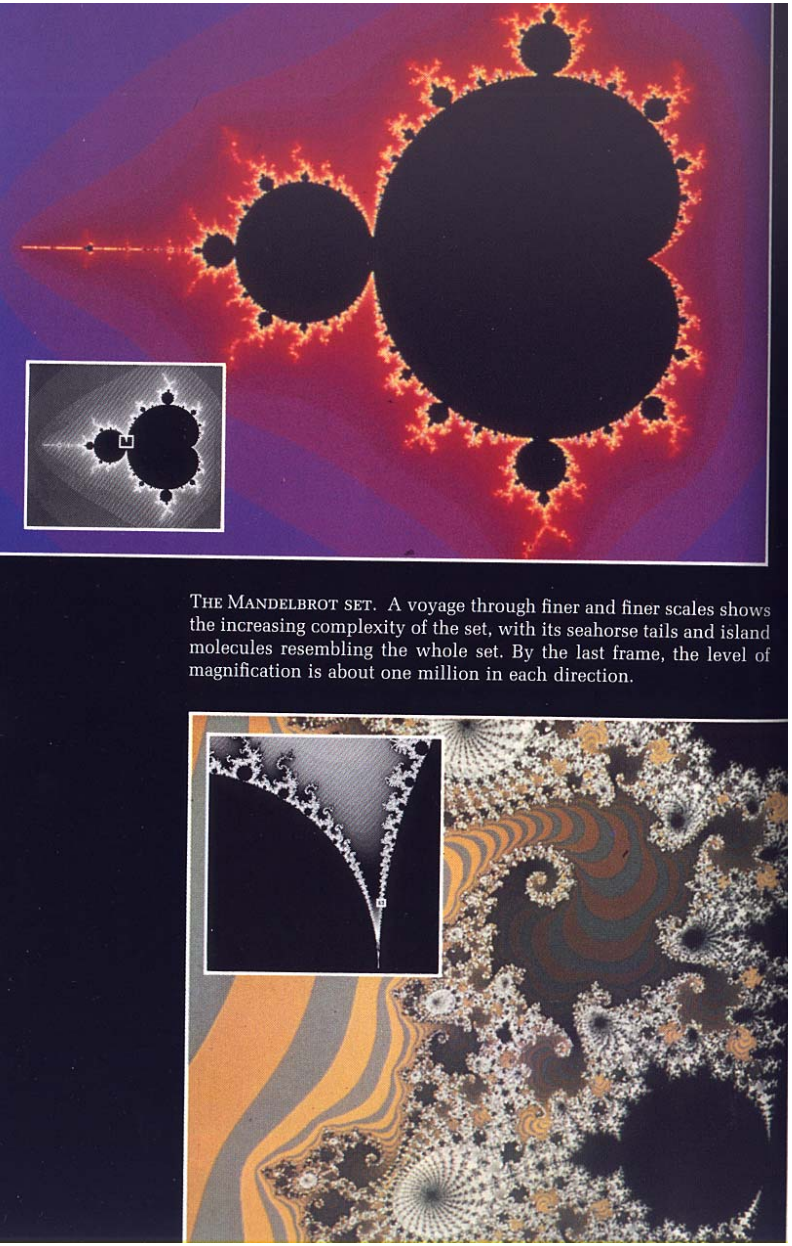
Computer images are a way of displaying fractals.

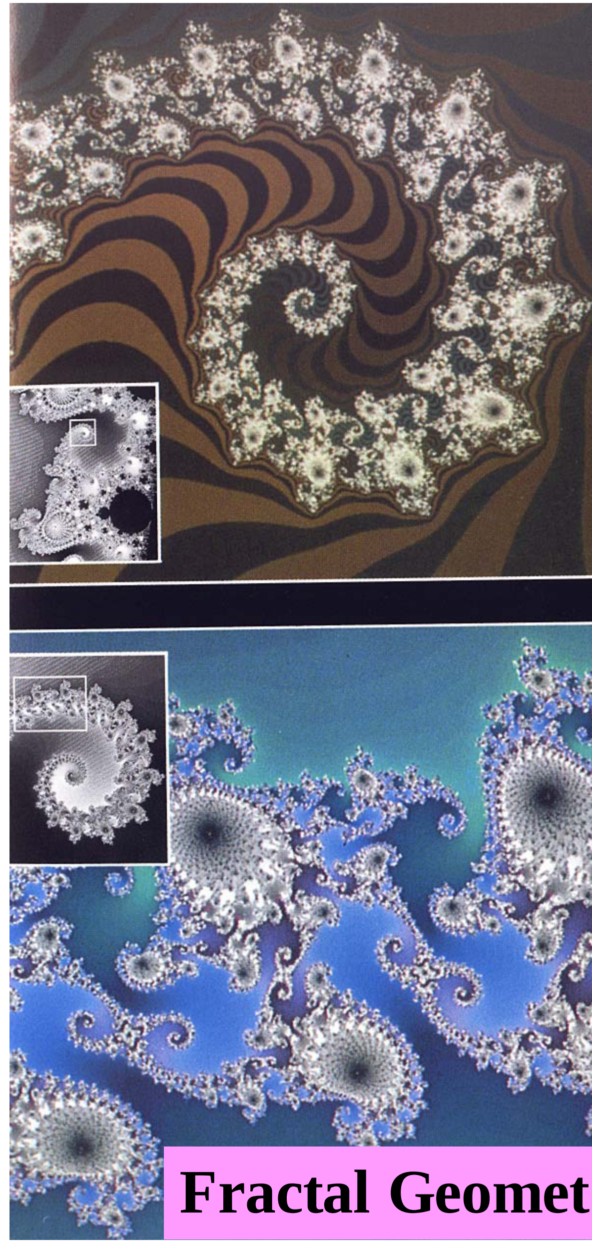
The Mandelbrot Set

Describe the Mandelbrot Set.

Why is it important?

The Mandelbrot Set





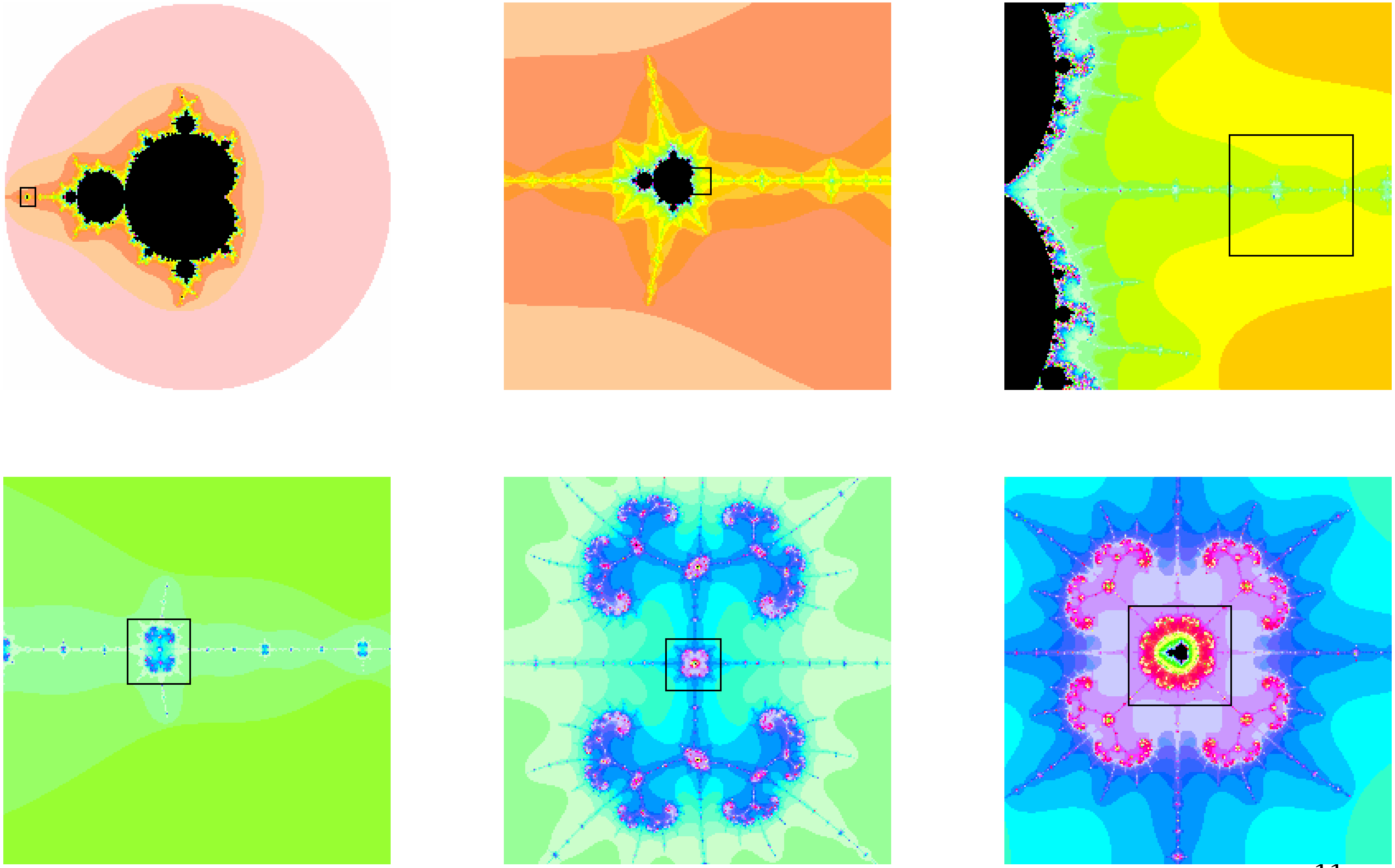
Fractal Geometry

Magnifications of the Mandelbrot set courtesy of Prof. Dr. Heinz-Otto Peitgen

From center insert of *Chaos*, James Gleick, Penguin Books, New York, 1987

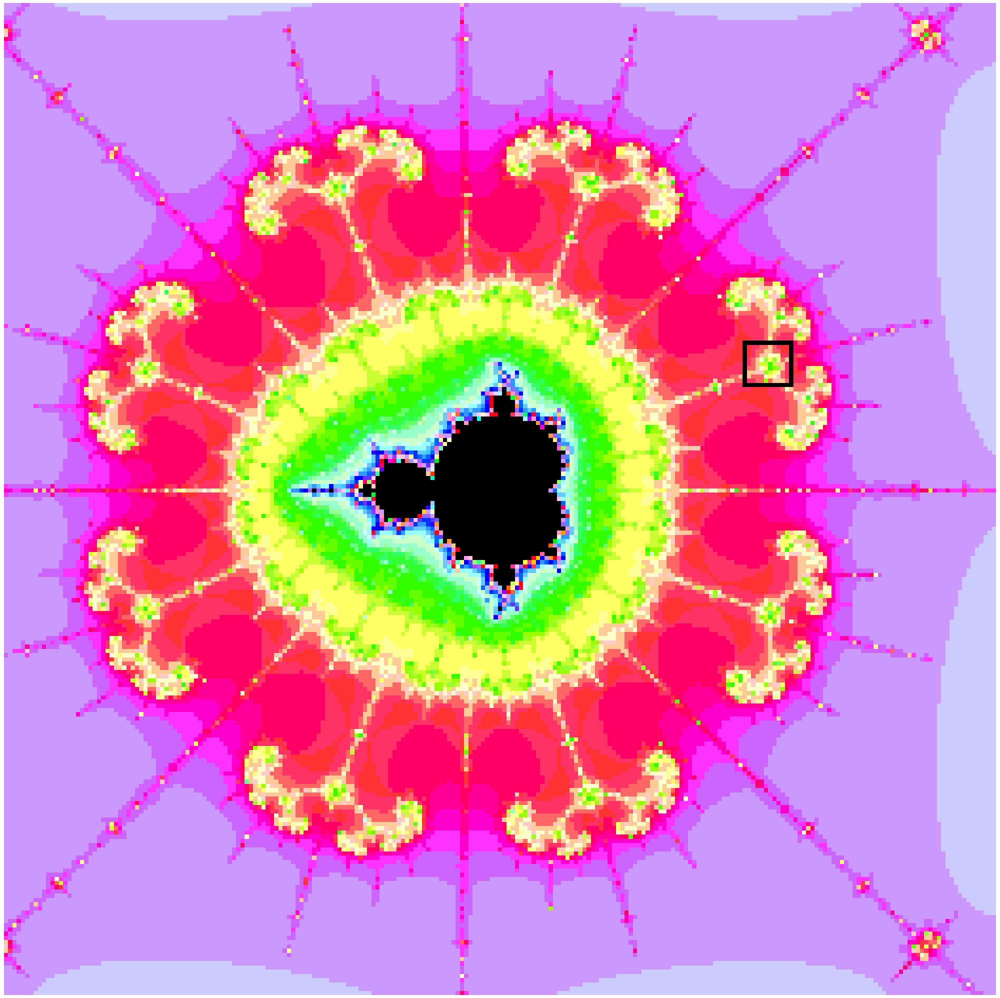
The Mandelbrot Set

Zoom in on the square.



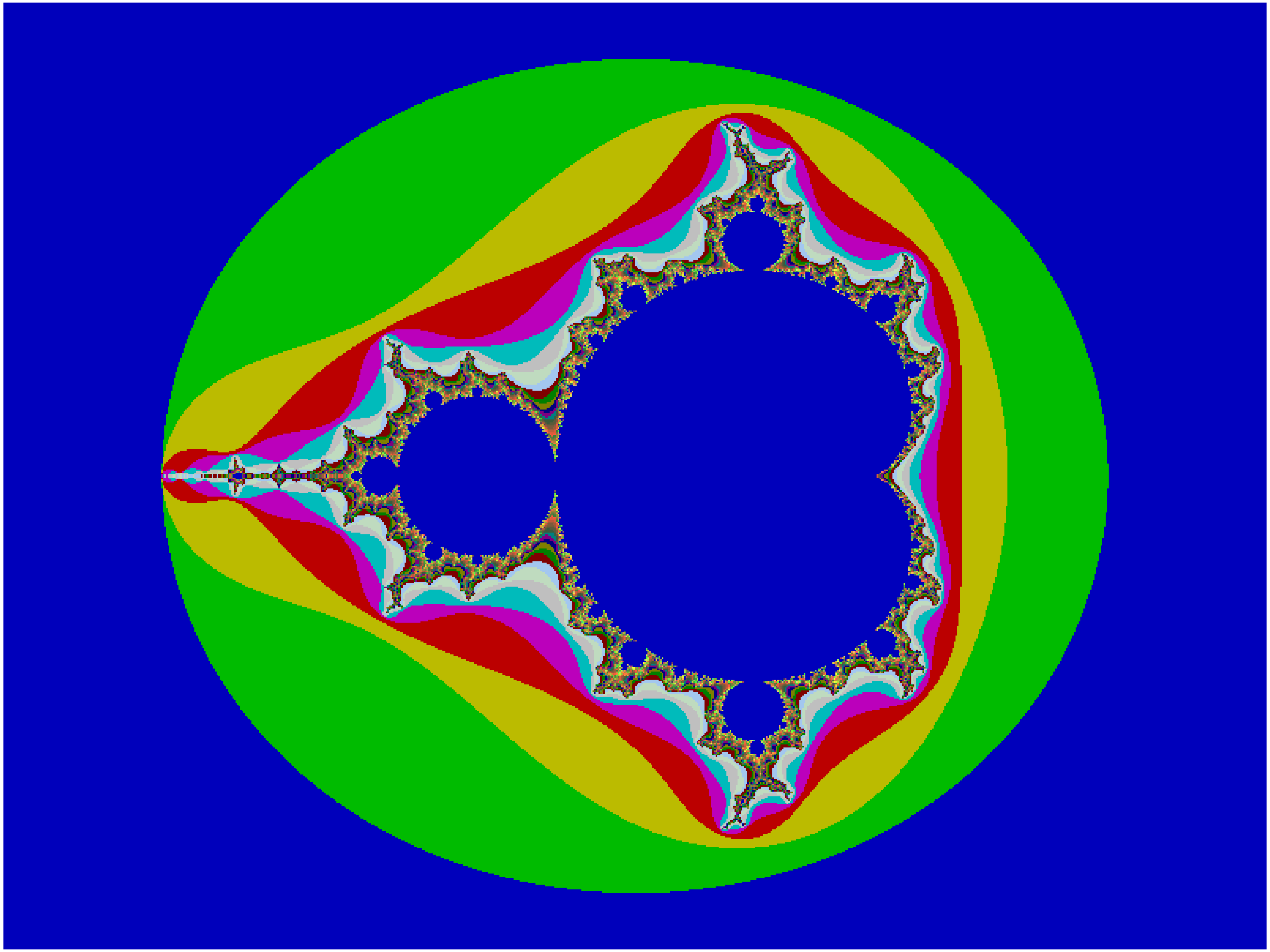
Slide from presentation of Thomas McGrath: Fab Fractal Frenzy, Fractal Geometry For Girls (FG)^2, Gateway Community College, North Haven Campus, June 2, 2006. Slides created using the free Fractint Fractal Generation program by the Stone Soup Group at Cornell University

The Mandelbrot Set



Slide from presentation of Thomas McGrath: Fab Fractal Frenzy**,** Fractal Geometry For Girls $(FG)^2$, Gateway Community College, North Haven Campus, June 2, 2006. Slide created using the free Fractint Fractal Generation program by the Stone Soup Group at Cornell University

The Mandelbrot Set



Slide from presentation of Thomas McGrath: Fab Fractal Frenzy**,** Fractal Geometry For Girls $(FG)^2$,
Gateway Community College, North Haven Campus, June 2, 2006. Slide created using the free Fractint Fractal Generation program by the Stone Soup Group at Cornell University.

What is self-similarity?

Explain how self-similarity can be found in the Koch curve.

Example of a Fractal

Koch Curve : Take a moment and remember how the curve is constructed. How many stages did you sketch?

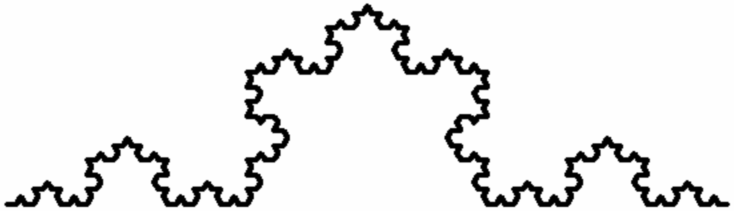
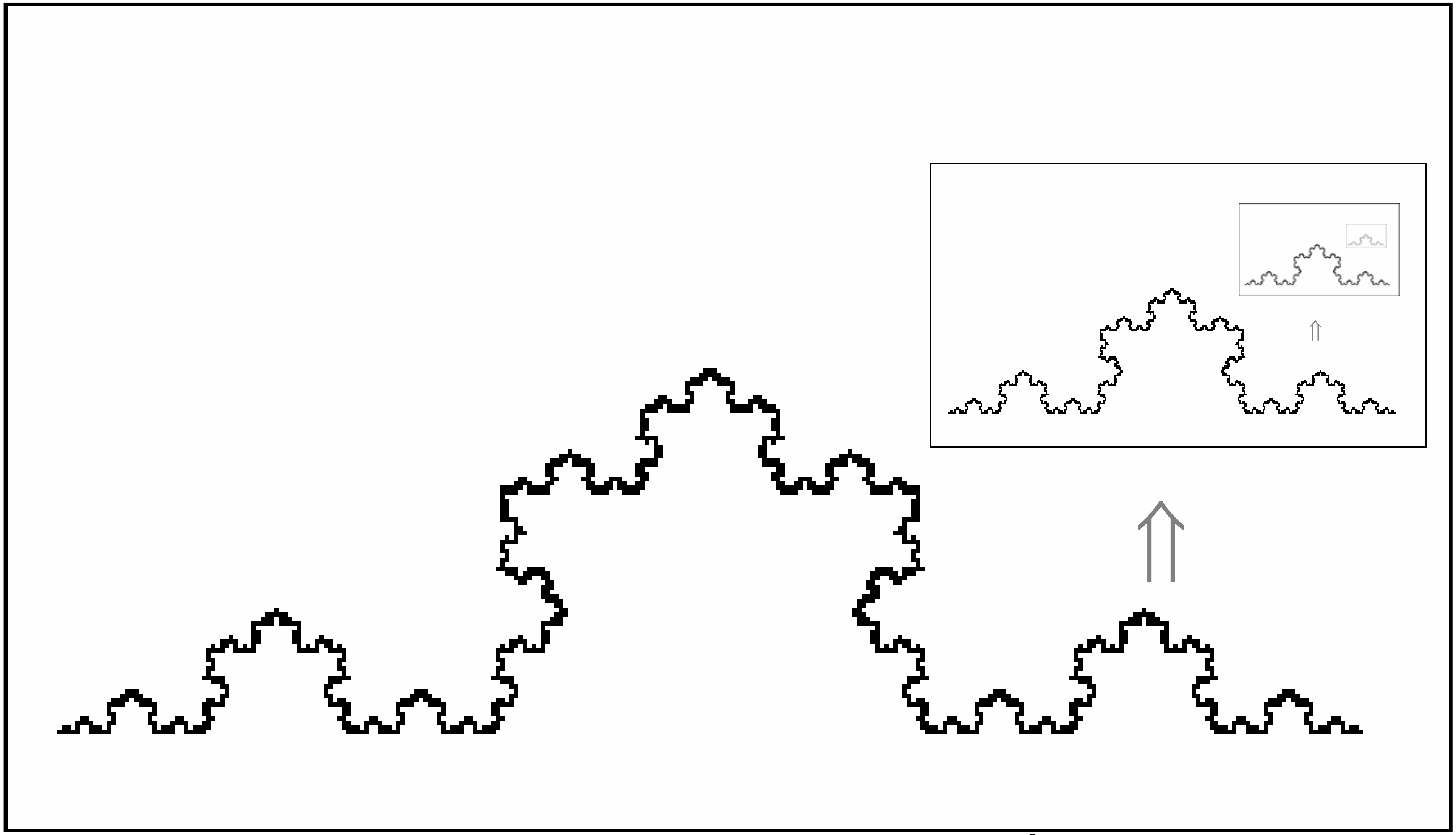


Illustration drawn in Logo: Courtesy of Ginny Jones, 3/28/2008
retired lecturer from Central Connecticut State University

Self-similarity

Koch Curve

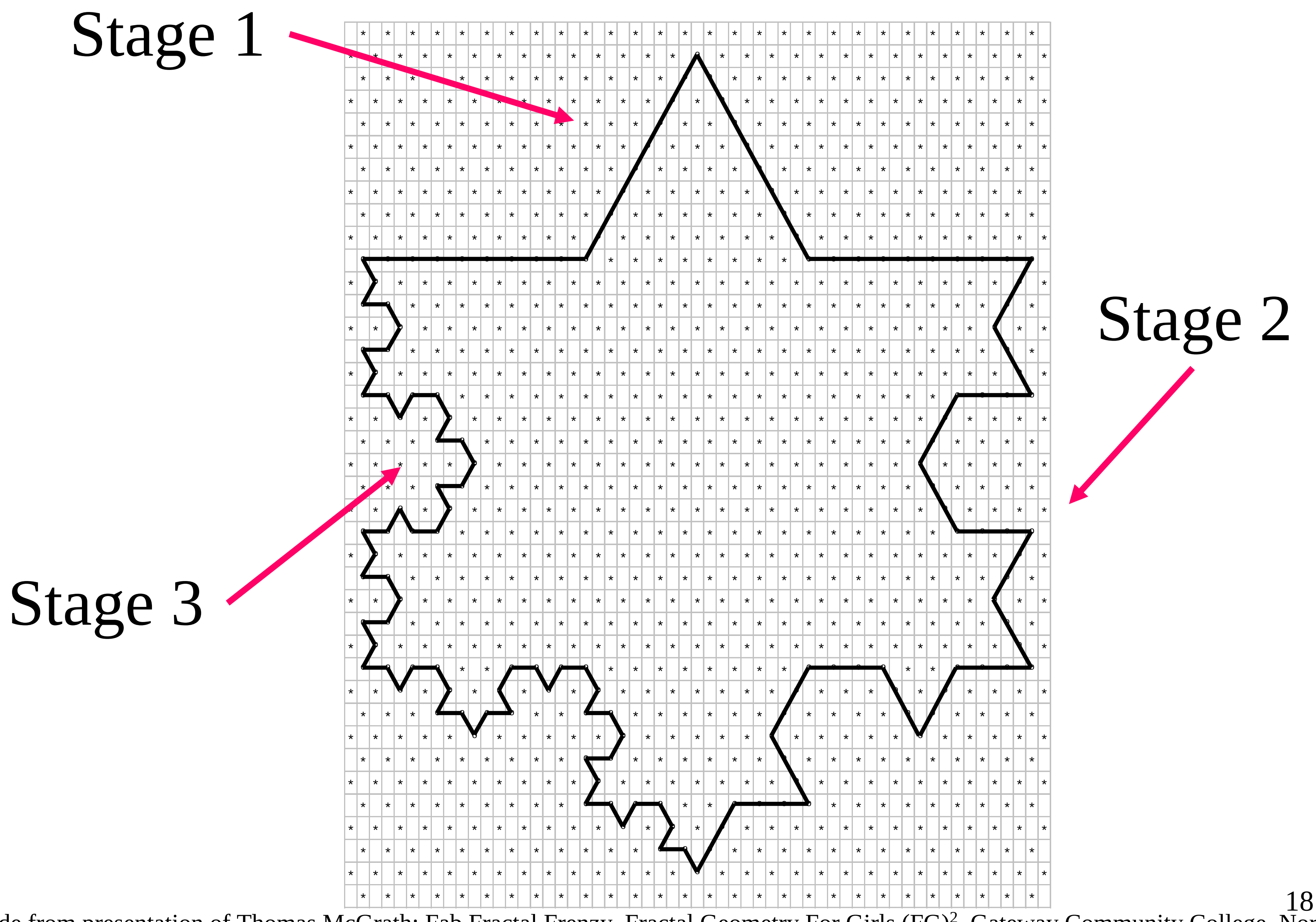


Slide from presentation of Thomas McGrath: Fab Fractal Frenzy, Fractal Geometry For Girls (FG)[2], Gateway Community College, North Haven Campus, June 2, 2006. Slides from The fractal geometry Web site, http://classes.yale.edu/fractals/ of Michael Frame, Benoit Mandelbrot and Nial Neger. Courtesy of Michael Frame.

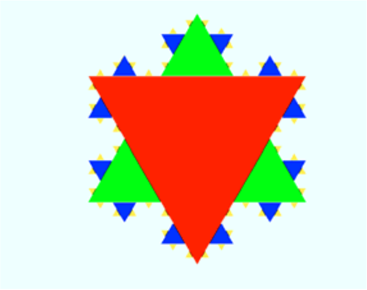
Koch Curve

- Self-similar
- At each stage in its construction, the length of the curve increases by a factor of 4/3.
- The resulting figure has infinite length in a finite area of the plane without intersecting itself.
- The curve is more than a line (not 1 D) and yet has no breadth (not 2D).

Koch Curve



Slide from presentation of Thomas McGrath: Fab Fractal Frenzy, Fractal Geometry For Girls $(FG)^2$, Gateway Community College, North Haven Campus, June 2, 2006.



Exercise: Koch Snowflake

1. With the specially designed and marked graph paper, create various stages of the Koch curve.
2. Color your “snowflake” to show the various levels of self-similarity. If the process continued indefinitely, the result would be called a Koch snowflake.

Illustration drawn in Logo: Courtesy of Ginny Jones, 3/28/2008

Koch Curve

		Koch Curve		Koch Snoflake	
Iteration	**Number of Sides**	**Side Length**	**Line Length**	**Perimeter**	**Area**
0	1	1.00	1.00	3.00	0.4330
1	4	3.33E-01	1.33	4.00	0.5774
2	16	1.11E-01	1.78	5.33	0.6415
3	64	3.70E-02	2.37	7.11	0.6700
4	256	1.23E-02	3.16	9.48	0.6827
5	1,024	4.12E-03	4.21	12.64	0.6883
6	4,096	1.37E-03	5.62	16.86	0.6908
7	16,384	4.57E-04	7.49	22.47	0.6919
8	65,536	***1.52E-04***	9.99	29.97	0.6924
9	262,144	5.08E-05	13.32	39.95	0.6926
10	1,048,576	1.69E-05	17.76	53.27	0.6927
11	4,194,304	5.65E-06	23.68	71.03	***0.6928***
12	16,777,216	1.88E-06	31.57	94.71	0.6928
13	67,108,864	6.27E-07	42.09	126.28	0.6928
14	268,435,456	**2.09E-07**	56.12	168.37	0.6928
15	1,073,741,824	6.97E-08	74.83	224.49	0.6928
17	17,179,869,184	7.74E-09	133.03	**399.10**	0.6928
20	1,099,511,627,776	2.87E-10	315.34	946.01	0.6928
22	17,592,186,044,416	3.19E-11	560.60	**1681.80**	0.6928
50	1.E+30	1.39E-24	1.77E+06	5.30E+06	0.6928
100	2.E+60	1.94E-48	3.12E+12	9.35E+12	0.6928

Slide from presentation of Thomas McGrath: Fab Fractal Frenzy, Fractal Geometry For Girls $(FG)^2$, Gateway Community College, North Haven Campus, June 2, 2006.

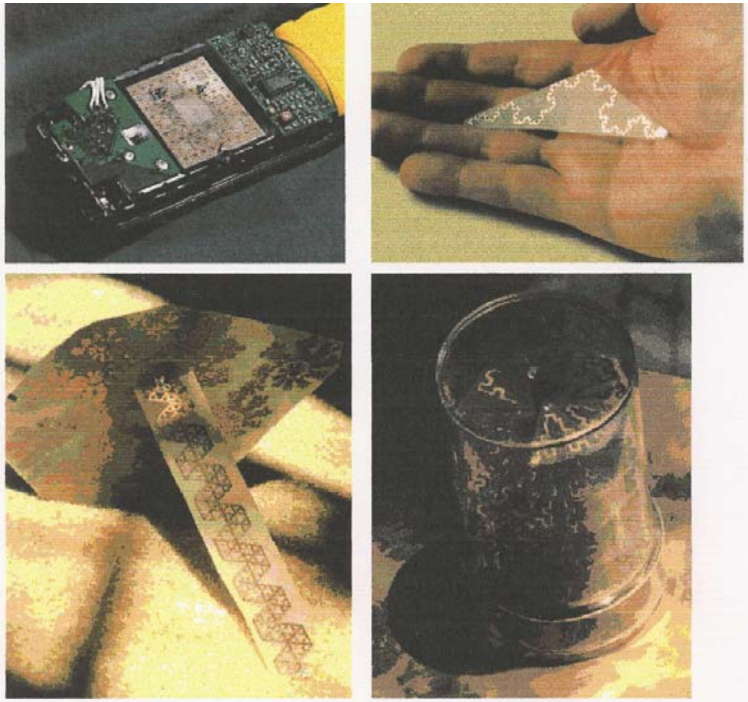
Fractal Dimension

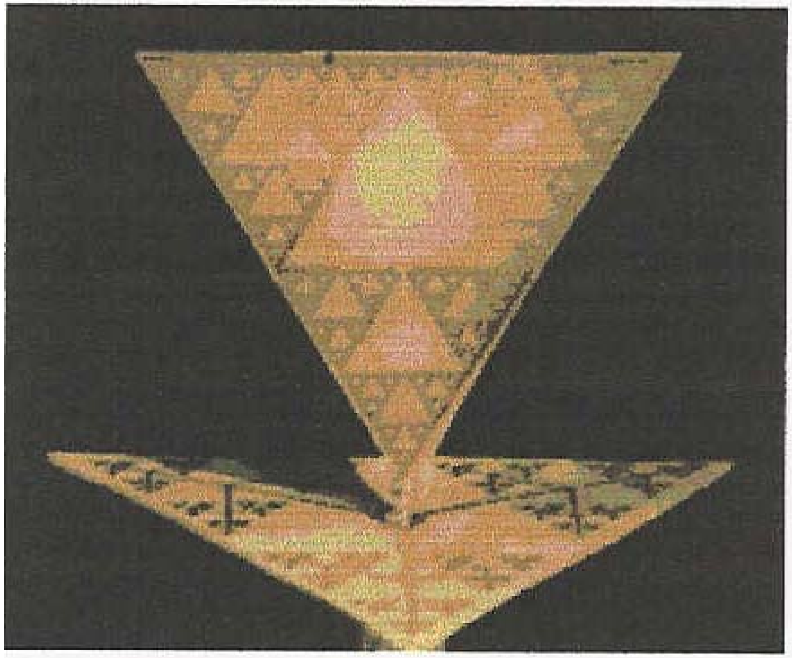
The Koch curve has a dimension of 1.26 --more than 1 but not 2.

Fractal Dimension: measures the degree of irregularity/roughness regardless of how much we zoom in on the curve.

Fractal Antennas

Fractals are important in building new devices. Here is a fractal antenna.

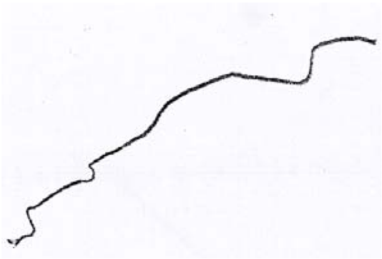




Fractal Antenna' Parts Photo Credit: Nathan Cohen © Fractal Antenna Systems, Inc. Used by Permission.

Reference: *Natural and Manufactured Fractals* from the fractal geometry web site, http://classes.yale.edu/fractals/ of Michael Frame, Benoit Mandelbrot and Nial Neger.

Fractal Coastlines



South Africa

Fractal dimension = 1*

Great Britain

Fractal dimension = 1.25*

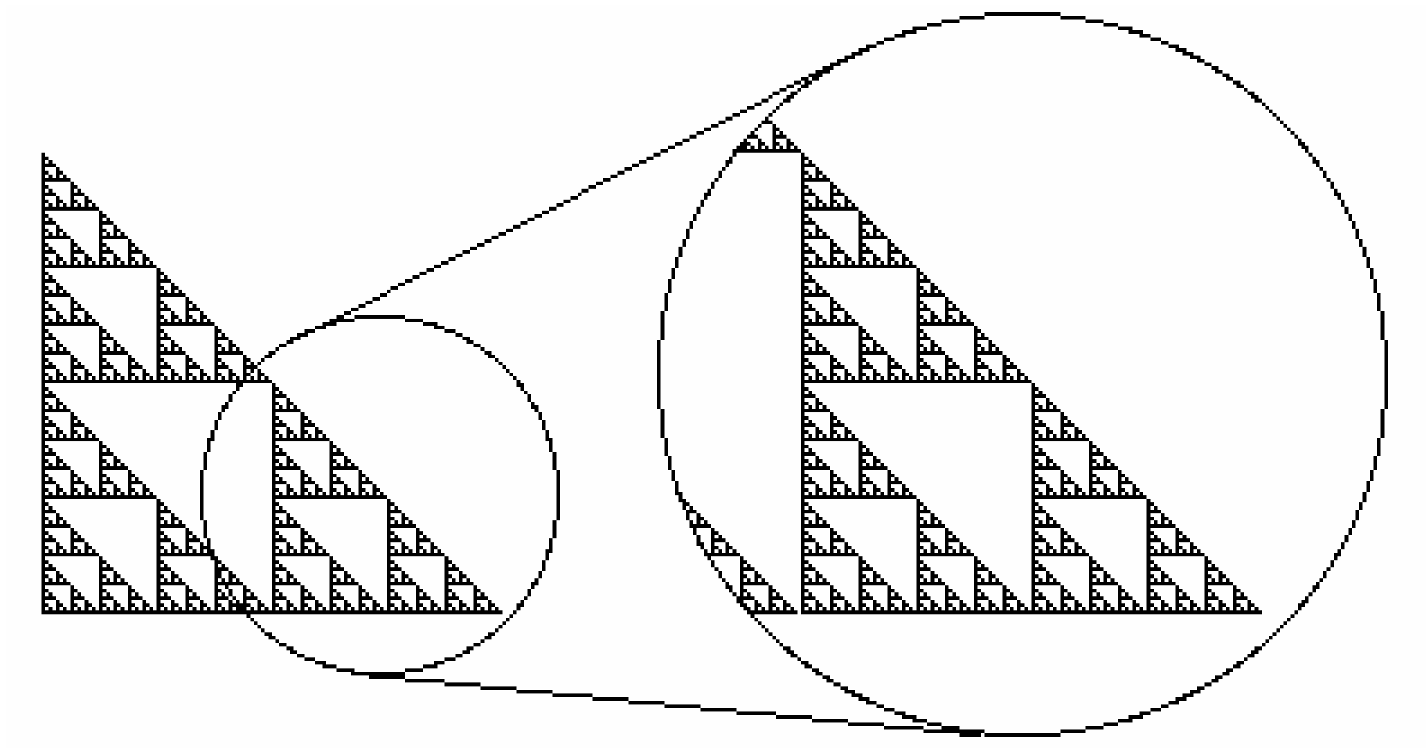
Norway

Fractal dimension = 1.52*

Notice that as the fractal dimension increases, the coastline is rougher.

* Dimensions as reported in Eglash, Ron. *African Fractals: Modern Computing and Indigenous Design*. New Brunswick, NJ: Rutgers University Press, 1999. 15. Hand calculations led to the following results: South Africa from Hotagterslip to southeast of Heidelberg: close to 1, Great Britain in the Holyhead region: 1.2 and Norway from south of Namsos to Nesna: 1.5.

The Sierpinski Triangle



Iterating this process produces, in the limit, the Sierpinski Gasket.

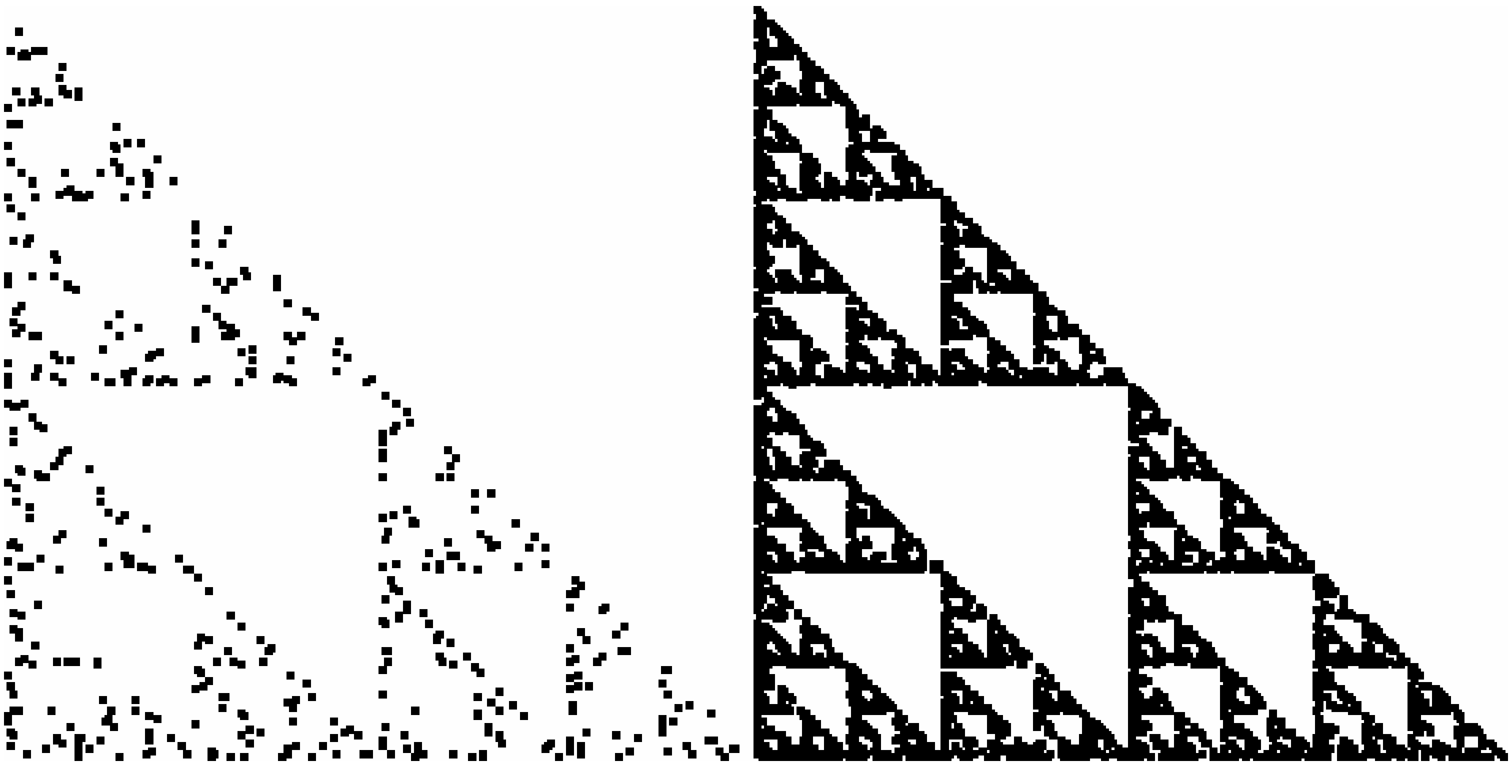
The gasket is self-similar, made up of smaller copies of itself.

Slide from presentation of Thomas McGrath: Fab Fractal Frenzy**,** Fractal Geometry For Girls (FG)[2], Gateway Community College, North Haven Campus, June 2, 2006. Reference from the fractal geometry Web site, http://classes.yale.edu/fractals/ of Michael Frame, Benoit Mandelbrot and Nial Neger. Courtesy of Michael Frame.

The Chaos Game

Roll die: 1,6 = **T**op; 2,5 = **L**eft; 3,4 = **R**ight

Mark a new point halfway to the corner (**T,L,R**) from old point.



The left picture shows 500 points, the right 5000.

Slide from presentation of Thomas McGrath: Fab Fractal Frenzy**,** Fractal Geometry For Girls $(FG)^2$, Gateway Community College, North Haven Campus, June 2, 2006. Slide from the fractal geometry Web site, http://classes.yale.edu/fractals/ of Michael Frame, Benoit Mandelbrot and Nial Neger. Courtesy of Michael Frame.

The Chaos Game: An Overlay Experiment

- Distribute dice and a triangle transparency to each student.
- Each student will roll the dice 20 times and carefully mark the points.
- Then overlay all of the transparencies.

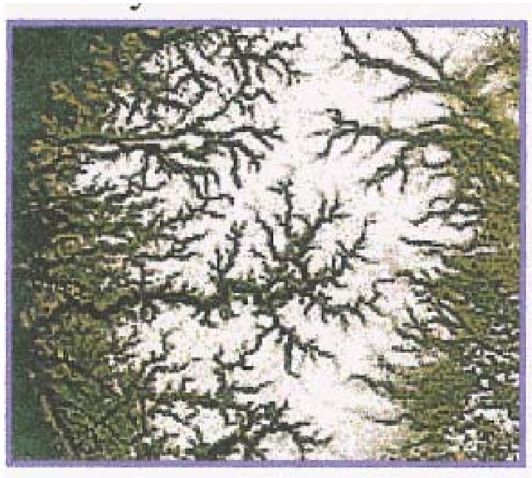
II Comparison of Classical and Fractal Geometry

Euclidean Geometry	**Fractal Geometry**
• Traditional/over 2000 yrs old	• modern discovery
• based on characteristic size or scale	• based on shapes being self-similar
• suits people-made objects	• describes shapes of nature
• usually described by formulas A = 1/2bh, P = 2l + 2w, etc.	• usually based on a building process that gets repeated and repeated

Comparison based on the chart presented in: Barnsley, Devaney, Mandelbrot, Peitgen, Saupe and Voss. *The Science of Fractal Images*. New York: Springer-Verlag, 1988. 26. Courtesy of Prof. Dr. Heinz-Otto Peitgen.

Fractals in Nature

Rivers and waterfalls can be fractal too! Notice the branching.



Clip Art

Reference and above illustration: Michael Frame, *Natural and Manufactured Fractals*, http://classes.yale.edu/fractals/ . Courtesy of Michael Frame.

Fractals in the Human Body

"Blood vessels must perform a bit of dimensional magic. Just as the Koch curve, for example, squeezes a line of infinite length into a small area, the circulatory system must squeeze a huge surface area into a limited volume. The fractal structure that nature has devised works so efficiently that, in most tissue, **no cell is ever more than three or four cells away from a blood vessel. Yet the vessels and blood take up little space, no more than five percent of the body."**

Reference: Gleick, James. *Chaos, Making A New Science. New York:* Penguin Books, 1987. 108.

Fractals in the Human Body

Lungs: “The lungs, too, need to pack the greatest possible surface into the smallest space. An animal’s ability to absorb oxygen is roughly proportional to the surface area of its lungs. **Typical human lungs pack in a surface bigger than a tennis court.”**

Reference: Gleick, James. *Chaos*, *Making A New Science*. *New York:* Penguin Books, 1987. 108.

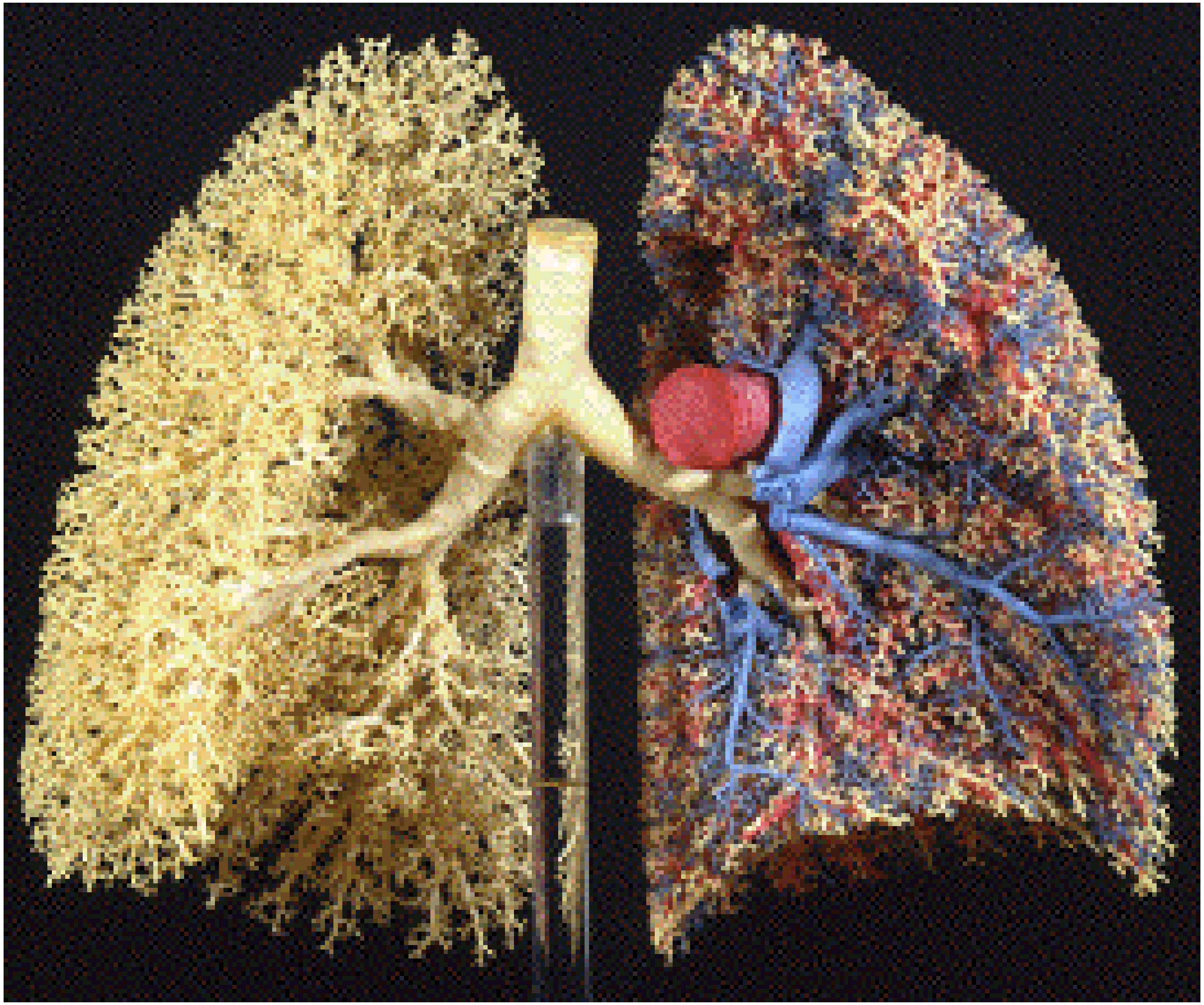


Image of human lung cast courtesy of Prof. Ewald R. Weibel, MD, DSc.

Reference: *Natural and Manufactured Fractals* from the fractal geometry Web site, http://classes.yale.edu/fractals/ of Michael Frame, Benoit Mandelbrot and Nial Neger.

Slide from presentation of Thomas McGrath: Fractals in BMET -- Fractal Models for Diagnosis and Design, Biomedical Symposium, Gateway Community College, May 11, 2006.

Fractals in the Human Body

"The body is filled with such [fractal] complexity. In the **digestive tract**, tissue reveals undulations within undulations"

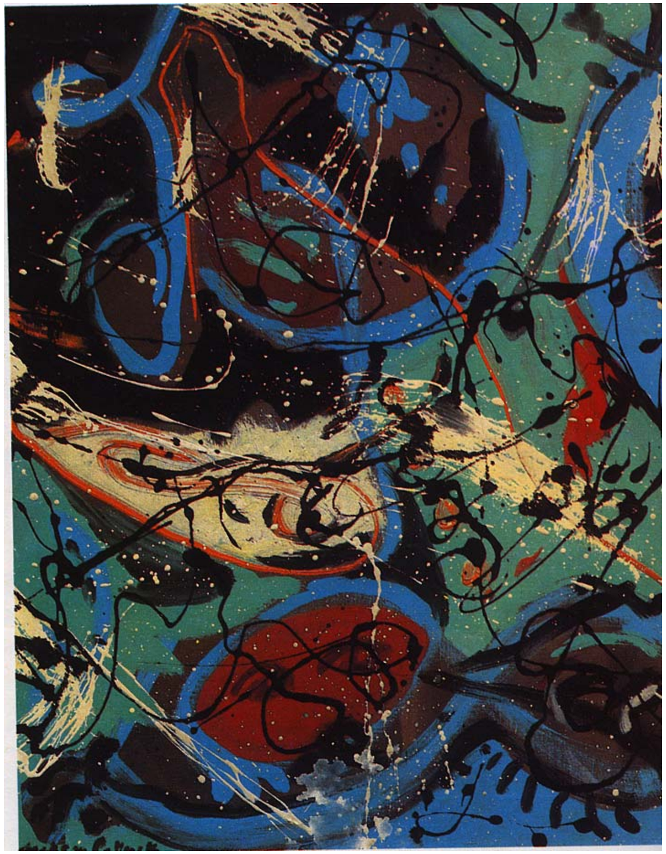
Reference: Gleick, James. *Chaos, Making A New Science. New York:* Penguin Books, 1987. 108-9.

Fractals in the Human Body

“ **bronchial** branching ... a fractal description turned out to fit the data.”

“The **urinary collection system** proved fractal.”

Reference: Gleick, James. *Chaos*, *Making A New Science*. *New York:* Penguin Books, 1987. 108-9.



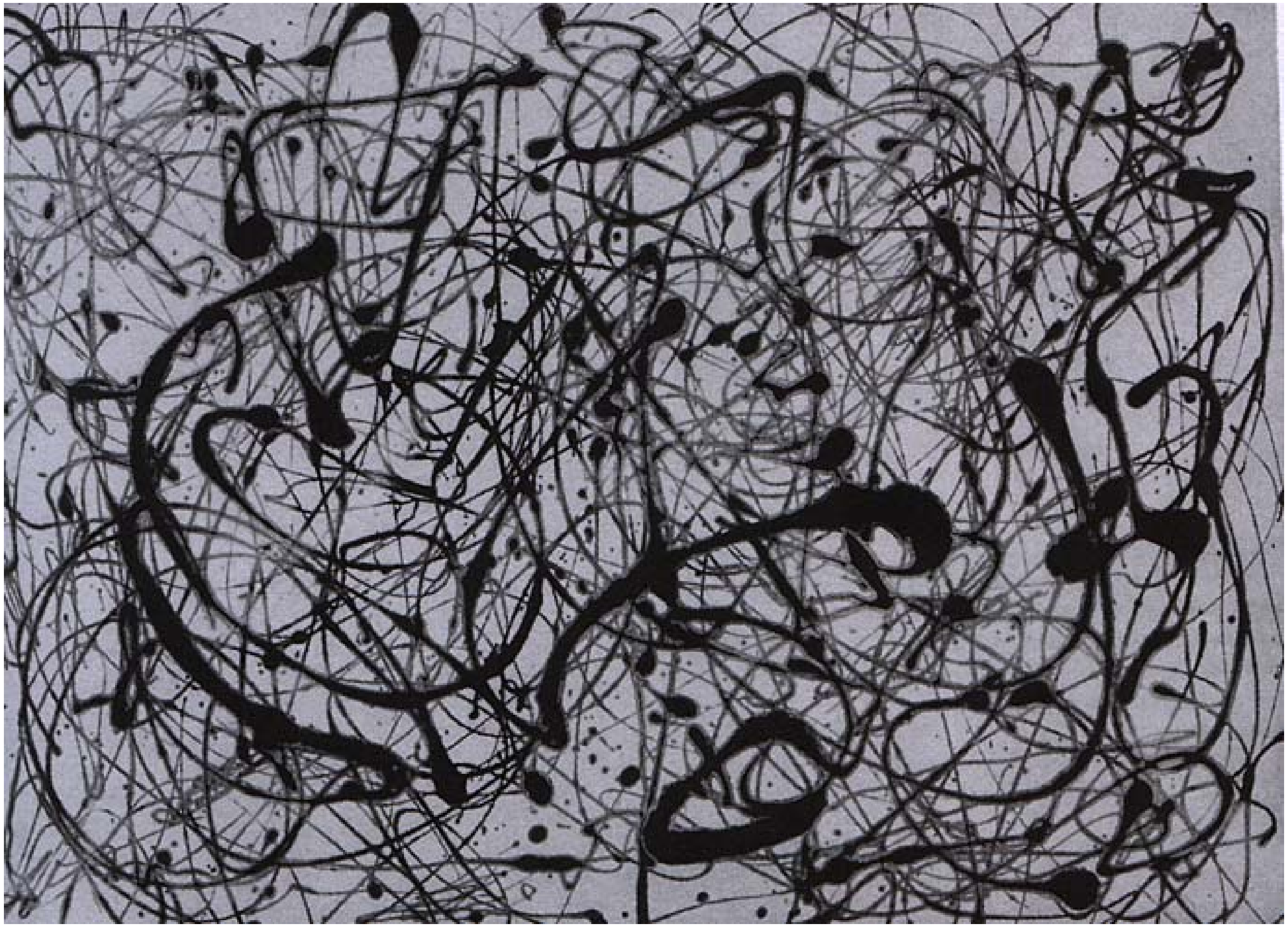
Fractal Dimensions determined by Taylor, Micolich and Jonas using "Box Counting" method

Fractal Dimension: close to 1

"Composition with Pouring II" (1943) by Jackson Pollock

Pollock, Jackson. *Composition with Pouring II* 1943, Oil and enamel paint on canvas, 25 x 22 1/8 in. (63.5 cm x 56.2 cm.) Hirshhorn Museum and Sculpture Garden, Smithsonian Institution, Washington DC

Reference: Taylor, "Fractal Expressionism", On-line: Internet, available at http://plus.maths.org/issue11/features/physics_world/index.html , page 4



“Number 14, 1948” by Jackson Pollock
Fractal Dimension: 1.45

Pollock, Jackson. *Number 14, 1948: Grey,* 1948: Grey. Oil and enamel paint on canvas, 25 x 22 1/8 in. (63.5 cm x 56.2 cm.) Hirshhorn Museum and Sculpture Garden, Smithsonian Institution, Washington DC

Reference: Taylor, “Fractal Expressionism”, On-line: Internet, available at http://plus.maths.org/issue11/features/physics_world/index.html , Fractals occur in art too! Here is a painting of Jackson Pollock who sometimes dripped paint on the canvas laid at his feet.

Fractal Dimension: 1.67

“Autumn Rhythm: Number 30” (1950)

by Jackson Pollock

Pollock, Jackson. *Autumn Rhythm: Number 30*, 1950, Oil on canvas, 8 ft. 10 1/2 in. x 17 ft. 8 in. (270.5 x 538.4 cm.) Collection: Metropolitan Museum of Art, George A. Hearn Fund, 1957.

Reference: Taylor, “Fractal Expressionism”, On-line: Internet, available at http/ ://materialscience.uoregon.edu/fractal_taylor.html, pp. 4-5. Currently available at http://plus.maths.org/issue11/features/physics_world/index.html .

"Blue Poles: Number 11" (1952) by Jackson Pollock
Fractal Dimension: 1.72

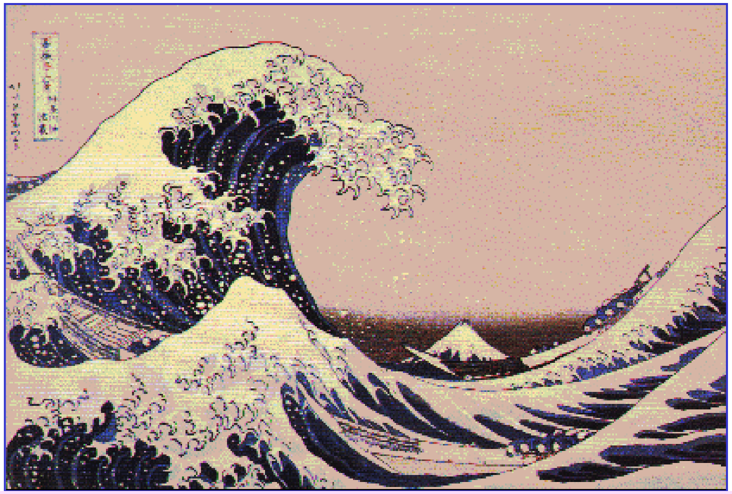
Pollock, Jackson. *Blue Poles: Number 11*, 1952, Enamel and aluminum paint with glass on canvas, 6 ft. 11 in. x 16 feet (210.8 x 487.6 cm.) Collection: Australian National Gallery, Canberra, Australia

Reference: Taylor, "Fractal Expressionism", On-line: Internet, available at http/ ://materialscience.uoregon.edu/fractal_taylor.html, page 5. Currently available at http://plus.maths.org/issue11/features/physics_world/index.html .

Fractal Art

Name two other artists that used fractals in their art?

Swamp Angel by Max Ernst. 1940. 26 ¼ X 32 ½ inches. Collection Kenneth McPerson, Rome. © 2008 Artists Rights Society, New York / ADAGP, Paris



Hokusai Painting: Michael Frame, *Fractal Geometry*, Yale University, " Natural and "Fractal Geometry Panorama", http://classes.yale.edu/Fractals//welcome.html . Date accessed: October 8, 2006.

Some Final Questions:

1. What did you like best about today's program?
2. What one new thing did you learn today?
3. What questions do you have?
4. Name a food that is fractal and one that isn't.