Introduction to Fractal Geometry and its Applications

College Visit

Final (Three Hour) Workshop

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Middle School Slides
Fractals

- Fractals can be described as
  - Broken
  - Fragmented
  - Irregular
- Concept created by Benoit Mandelbrot to describe nature and measure roughness.

Picture of Benoit B. Mandelbrot was taken at his lecture at Worcester Polytechnic Institute, November 2006 and the picture of the Mandelbrot set is from: The fractal geometry Web site, [http://classes.yale.edu/fractals/](http://classes.yale.edu/fractals/) of Michael Frame, Benoit Mandelbrot and Nial Neger. Courtesy of Michael Frame.
“I coined fractal from the Latin adjective fractus. The corresponding Latin verb frangere means ‘to break:’ to create irregular fragments.” Benoit B. Mandelbrot

This is a shoal near the coastline of the Bahamas. It is very jagged and rough.

Regardless of the scale, the actual coastlines appear to have the same amount of jaggedness.

Geometry of Nature

Cezanne’s statement about painting: “Everything in Nature can be viewed in terms of cones, cylinders and spheres.” Can you find a cone shape? A cylinder?

Reference: Michael Frame, Natural and Manufactured Fractals, http://classes.yale.edu/fractals/
1960 – 1990s Mandelbrot at IBM Research

NSF Geometry Super Computer Project

“Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in straight lines.”

Mandelbrot

Today

Chaos and Dynamical Systems are current fields of research.

Computer images are a way of displaying fractals.
The Mandelbrot Set

Describe the Mandelbrot Set.

Why is it important?

Magnifications of the Mandelbrot set courtesy of Prof. Dr. Heinz-Otto Peitgen.
The Mandelbrot Set

Zoom in on the square.

Slide from presentation of Thomas McGrath: Fab Fractal Frenzy, Fractal Geometry For Girls (FG)², Gateway Community College, North Haven Campus, June 2, 2006. Slides created using the free Fractint Fractal Generation program by the Stone Soup Group at Cornell University
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What is self-similarity?

Explain how self-similarity can be found in the Koch curve.
Example of a Fractal

Koch Curve: Take a moment and remember how the curve is constructed. How many stages did you sketch?

Illustration drawn in Logo: Courtesy of Ginny Jones, 3/28/2008 retired lecturer from Central Connecticut State University
Self-similarity

Koch Curve

Koch Curve

- Self-similar

- At each stage in its construction, the length of the curve increases by a factor of 4/3.

- The resulting figure has infinite length in a finite area of the plane without intersecting itself.

- The curve is more than a line (not 1 D) and yet has no breadth (not 2D).
Exercise: Koch Snowflake

1. With the specially designed and marked graph paper, create various stages of the Koch curve.

2. Color your “snowflake” to show the various levels of self-similarity. If the process continued indefinitely, the result would be called a Koch snowflake.

Illustration drawn in Logo: Courtesy of Ginny Jones, 3/28/2008
## Koch Curve

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<th>Line Length</th>
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<th>Area</th>
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<td>3.00</td>
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<td>9.35E+12</td>
<td>0.6928</td>
</tr>
</tbody>
</table>

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Slide from presentation of Thomas McGrath: Fab Fractal Frenzy, Fractal Geometry For Girls (FG)², Gateway Community College, North Haven Campus, June 2, 2006.
The Koch curve has a dimension of 1.26 --more than 1 but not 2.

Fractal Dimension: measures the degree of irregularity/roughness regardless of how much we zoom in on the curve.
Fractal Antennas

Fractals are important in building new devices. Here is a fractal antenna.

Reference: *Natural and Manufactured Fractals* from the fractal geometry web site, [http://classes.yale.edu/fractals/](http://classes.yale.edu/fractals/) of Michael Frame, Benoit Mandelbrot and Nial Neger.
Fractal Coastlines

South Africa
Fractal dimension = 1*

Great Britain
Fractal dimension = 1.25*

Norway
Fractal dimension = 1.52*

Notice that as the fractal dimension increases, the coastline is rougher.

* Dimensions as reported in Eglash, Ron. *African Fractals: Modern Computing and Indigenous Design.* New Brunswick, NJ: Rutgers University Press, 1999. 15. Hand calculations led to the following results: South Africa from Hotagterslip to southeast of Heidelberg: close to 1, Great Britain in the Holyhead region: 1.2 and Norway from south of Namsos to Nesna: 1.5.
The Sierpinski Triangle

Iterating this process produces, in the limit, the Sierpinski Gasket.
The gasket is self-similar, made up of smaller copies of itself.

The Chaos Game

Roll die: 1,6 = Top; 2,5 = Left; 3,4 = Right

Mark a new point halfway to the corner (T,L,R) from old point.

The left picture shows 500 points, the right 5000.

The Chaos Game: An Overlay Experiment

- Distribute dice and a triangle transparency to each student.
- Each student will roll the dice 20 times and carefully mark the points.
- Then overlay all of the transparencies.
II  Comparison of Classical and Fractal Geometry

<table>
<thead>
<tr>
<th>Euclidean Geometry</th>
<th>Fractal Geometry</th>
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<tbody>
<tr>
<td>• Traditional/over 2000 yrs old</td>
<td>• modern discovery</td>
</tr>
<tr>
<td>• based on characteristic size or scale</td>
<td>• based on shapes being self-similar</td>
</tr>
<tr>
<td>• suits people-made objects</td>
<td>• describes shapes of nature</td>
</tr>
<tr>
<td>• usually described by formulas</td>
<td>• usually based on a building process</td>
</tr>
<tr>
<td>$A = \frac{1}{2}bh, \quad P = 2l + 2w$, etc.</td>
<td>that gets repeated and repeated</td>
</tr>
</tbody>
</table>

Fractals in Nature

Rivers and waterfalls can be fractal too! Notice the branching.

Fractals in the Human Body

“Blood vessels must perform a bit of dimensional magic. Just as the Koch curve, for example,
squeezes a line of infinite length into a small area, the circulatory system must squeeze a huge surface area into a limited volume. The fractal structure that nature has devised works so efficiently that, in most tissue, no cell is ever more than three or four cells away from a blood vessel. Yet the vessels and blood take up little space, no more than five percent of the body.”

Lungs: “The lungs, too, need to pack the greatest possible surface into the smallest space. An animal’s ability to absorb oxygen is roughly proportional to the surface area of its lungs. **Typical human lungs pack in a surface bigger than a tennis court.**”

Image of human lung cast courtesy of Prof. Ewald R. Weibel, MD, DSc.

Reference: *Natural and Manufactured Fractals* from the fractal geometry Web site, [http://classes.yale.edu/fractals/](http://classes.yale.edu/fractals/) of Michael Frame, Benoit Mandelbrot and Nial Neger.

Slide from presentation of Thomas McGrath: Fractals in BMET -- Fractal Models for Diagnosis and Design, Biomedical Symposium, Gateway Community College, May 11, 2006.
Fractals in the Human Body

“The body is filled with such [fractal] complexity. In the digestive tract, tissue reveals undulations within undulations”

Fractals in the Human Body

“bronchial branching ... a fractal description turned out to fit the data.”

“The urinary collection system proved fractal.”

Fractal Dimensions determined by Taylor, Micolich and Jonas using “Box Counting” method

Fractal Dimension: close to 1

“Composition with Pouring II” (1943) by Jackson Pollock

© 2008 The Pollock-Krasner Foundation / Artists Rights Society (ARS), New York

Pollock, Jackson. Composition with Pouring II 1943, Oil and enamel paint on canvas, 25 x 22 1/8 in. (63.5 cm x 56.2 cm.) Hirshhorn Museum and Sculpture Garden, Smithsonian Institution, Washington DC

“Number 14, 1948” by Jackson Pollock
Fractal Dimension: 1.45

© 2008 The Pollock-Krasner Foundation / Artists Rights Society (ARS), New York
Pollock, Jackson. Number 14, 1948: Grey, 1948: Grey. Oil and enamel paint on canvas, 25 x 22 1/8 in. (63.5 cm x 56.2 cm.) Hirshhorn Museum and Sculpture Garden, Smithsonian Institution, Washington DC

Fractals occur in art too! Here is a painting of Jackson Pollock who sometimes dripped paint on the canvas laid at his feet.

Fractal Dimension: 1.67

“Autumn Rhythm: Number 30” (1950) by Jackson Pollock

“Blue Poles: Number 11” (1952) by Jackson Pollock
Fractal Dimension: 1.72

© 2008 The Pollock-Krasner Foundation / Artists Rights Society (ARS), New York

Pollock, Jackson. *Blue Poles: Number 11*, 1952, Enamel and aluminum paint with glass on canvas, 6 ft. 11 in. x 16 feet (210.8 x 487.6 cm.) Collection: Australian National Gallery, Canberra, Australia

Name two other artists that used fractals in their art?


Some Final Questions:

1. What did you like best about today’s program?
2. What one new thing did you learn today?
3. What questions do you have?
4. Name a food that is fractal and one that isn’t.