Introduction to Fractal Geometry and its Applications

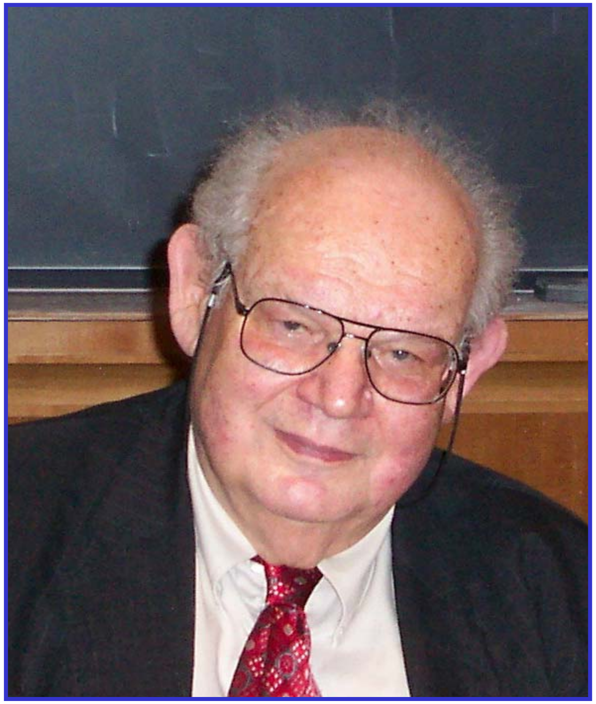
Workshop 7: Recursion

Middle School Slides: Fractal Math

Fractals

- Broken
- Fragmented
- Irregular

Concept created by Benoit Mandelbrot to describe the shapes of nature and to measure roughness.



Picture of Benoit B. Mandelbrot was taken at his lecture at Worcester Polytechnic Institute, November 2006 and the picture of the Mandelbrot set is from: The fractal geometry Web site, http://classes.yale.edu/fractals/ of Michael Frame, Benoit Mandelbrot and Nial Neger. Courtesy of Michael Frame.

Benoit B. Mandelbrot

Fibonacci Numbers (interesting but not fractal)

Consider the Fibonacci numbers starting with 1 and 1:

1, 1, 2, 3, 5, 8,…

After the first two Fibonacci numbers the next element is always the sum of the two previous Fibonacci numbers.

Notice that 5 = 2 + 3 and 8 = 3 + 5

What are the next five (and then six) Fibonacci numbers?

The Fibonacci Numbers

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …

Recursion and the Fibonacci Numbers

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, …

Recursion takes place when the next number of a set is a combination of previous elements.

Note: The Fibonacci numbers are “recursive but not fractal.”

Fibonacci Numbers in Flowers

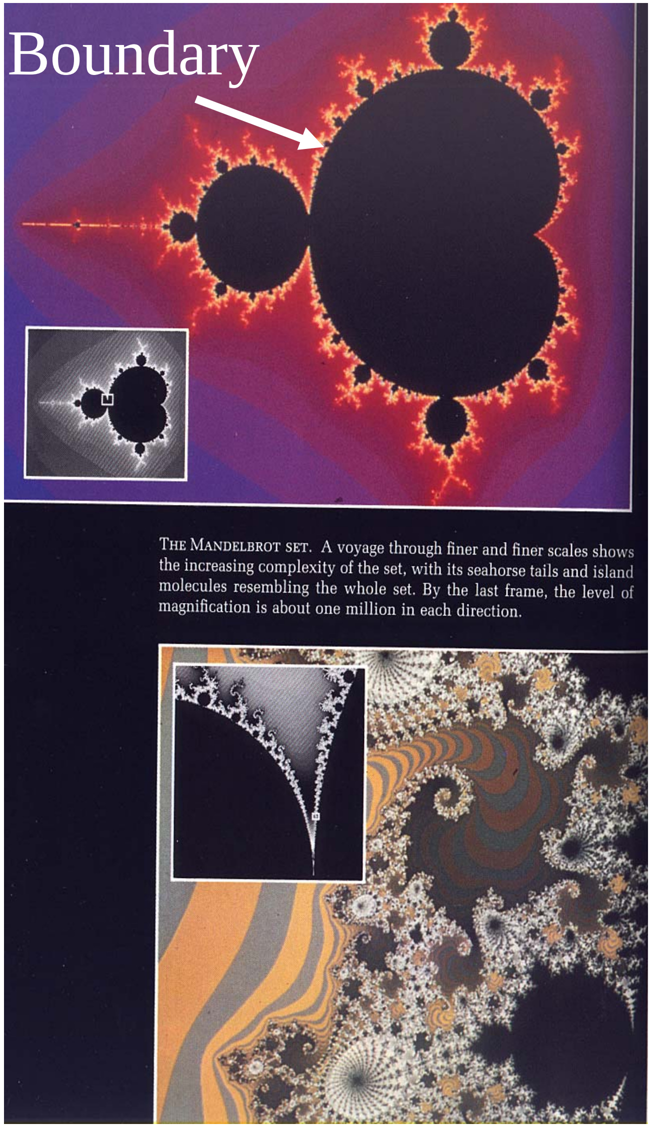
Number of Petals	Example of Flowers
89	Some Daises
55	Other Daises
5	Buttercups & Wild Rose
3	Lily

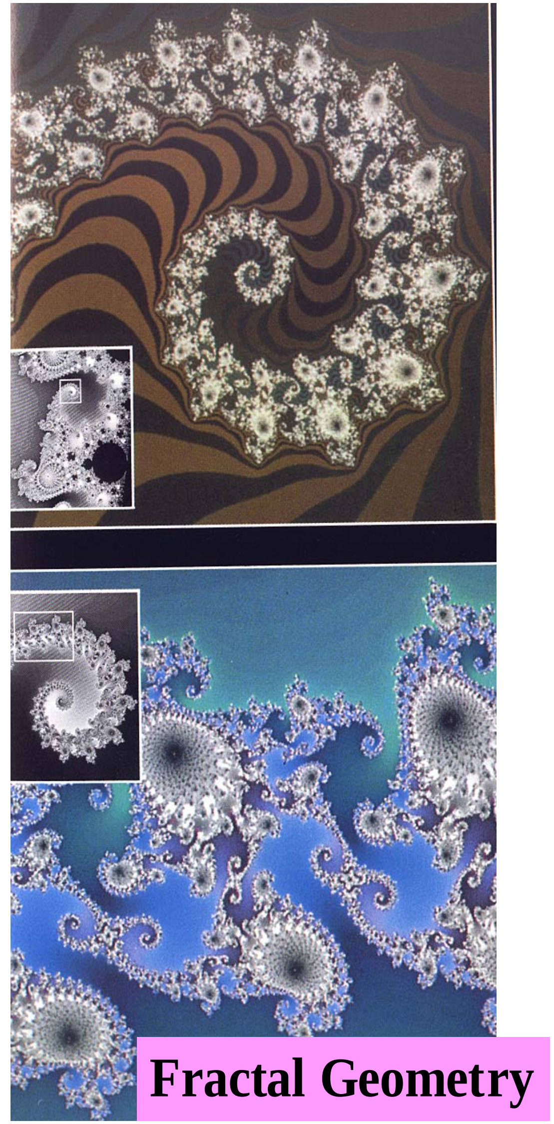
Reference: Slides of a previous MAT105: Introduction to Mathematical Thought student at Connecticut College

The Fibonacci Spiral

Count the number of white spiral paths and then the number of green spiral paths.

Illustrations of Fibonacci numbers in opposing pinecone spirals courtesy of Janet Hayes, Instructional Developer, Information Services, Connecticut College
Pinecone photograph: Levin, Eddy. "Golden Proportion." The Golden Proportion and Dental Aesthetics. <http://www.goldenmeangauge.co.uk/index.html> (1 May 2000). Courtesy of Eddy. Levin.





The Mandelbrot Set

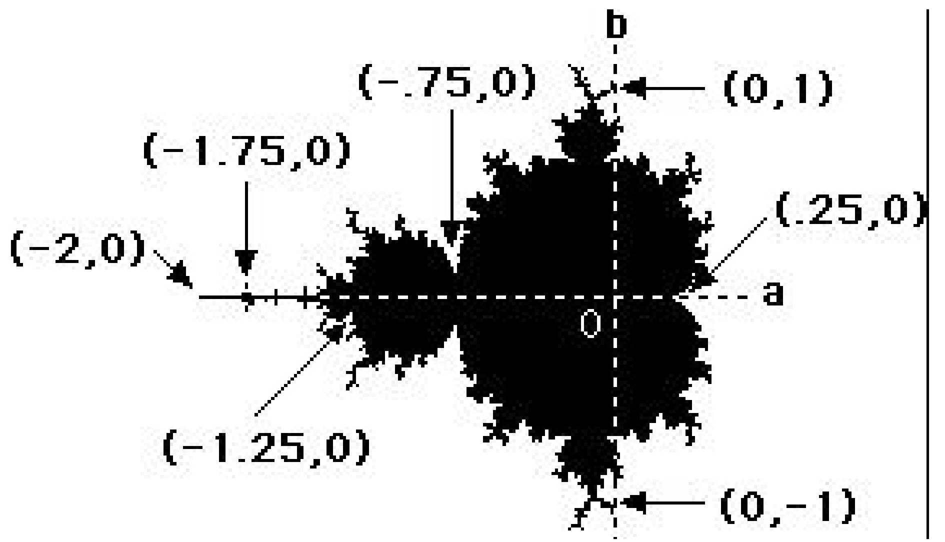
Magnifications of the Mandelbrot set courtesy of Prof. Dr. Heinz-Otto Peitgen

From center insert of *Chaos*, James Gleick, Penguin Books, New York, 1987

The Mandelbrot set is governed by a single equation and its boundary contains fractals!

The Mandelbrot Set is governed by one simple equation:
$z \Leftrightarrow z^2 + C$ where z and C are complex numbers.

You will study complex numbers in high school



Mandelbrot Illustration: Courtesy of Prof. Michael Frame, Mathematics Department, Yale University

The Mandelbrot Set (optional slide)

Example 1: Let $c = 1$
with the initial condition $x_0 = 0$
Simplified Mandelbrot:
$x_1 = x_0^2 + c$
$x_1 = 0^2 + 1 = 1$
$x_2 = x_1^2 + c$
$x_1 = 1^2 + 1 = 2$
$x_2 = x_1^2 + c$
$x_2 = 2^2 + 1 = 4 + 1 = 5$

Orbit: $x_0, x_1, x_2, x_3, \ldots = 0, 1, 2, 5, 26,$
$677, \ldots \rightarrow \infty$

Conclusion: $c = 1$ will not be in the Mandelbrot set.

The Mandelbrot Set and Recursion

Orbit for c = 1:

0, 1, 2, 5, 26, 677, …→ ∞

Recursion takes place when the next number of a set (here an orbit) is a combination of previous elements.

Note: The Mandelbrot set contains fractals and is recursive!

The Mandelbrot Set (optional slide)

Example 2: Let $c = -1$ and $x_0 = 0$
Simplified Mandelbrot:
$x_1 = x_0^2 + c$
$x_1 = 0^2 + (-1) = -1$
$x_2 = x_1^2 + c$
$x_1 = (-1)^2 + -1 = 1 + -1 = 0$
$x_2 = x_1^2 + c$
$x_2 = 0^2 + (-1) = 0 - 1 = -1$

Notice the orbit has values that alternate

Orbit: $x_0, x_1, x_2, x_3, \ldots = 0, -1, 0, -1, \ldots$

Conclusion $c = -1$ will be in the Mandelbrot set since the orbit does not approach infinity.

The Mandelbrot Set (optional slide)

Example 3: Let $c = -2$ and $x_0 = 0$
Simplified Mandelbrot:
$x_1 = x_0{}^2 + c$
$x_1 = 0^2 + (-2) = -2$
$x_2 = x_1{}^2 + c$
$x_1 = (-2)^2 + -2 = 4 – 2 = 2$
$x_2 = x_1{}^2 + c$
$x_2 = (2)^2 + (-2) = 4 -2 = 2$
$x_3 = x_2{}^2 + c$
$x_2 = (2)^2 + (-2) = 4 -2 = 2$

Notice that after two iterations the orbit approaches a fixed number. Here that number is 2.

Orbit: $x_0, x_1, x_2, x_3, \ldots = 0, -2, 2, 2, 2, \ldots$

Conclusion c = -2 will be in the Mandelbrot set since the orbit does not approach infinity.

The Mandelbrot Set (optional slide)

The Mandelbrot set will contain all those values of c for which the orbits of $x_0 = 0$ (really z_0—the complex number) do not approach infinity.

Activity: Mandelbrot Explorer

Learn more about orbits of the Mandelbrot set by visiting Professor Robert Devaney's Mandelbrot Explorer at his website:

http://math.bu.edu/DYSYS/explorer/page1.html at the Mathematics Department at Boston University.

Preparing for the College Program

Have a pizza and determine who will prepare the following topics:

1. The life and work of Prof. Mandelbrot
2. What is self-similarity?
3. Draw the Sierpinski triangle
4. Draw the Koch curve
5. Discuss where fractals are in the human body
6. Discuss where fractals are in the universe
7. Discuss fractal dimensions
8. Why are the drip paintings of Jackson Pollock fractal?
9. Why are fractals important?
10. Do a Google search for fractals.