Introduction to Fractal Geometry and its Applications

Workshop 5: Fractal Dimensions

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Middle School Slides: Fractal Math

Review of Workshop 4: Fractals in Nature and Science The Koch Curve

The Koch Curve

Can you describe it?

Remember each line segment is divided into 3 pieces (r = 1/3) resulting in 4 new self-similar pieces N = 4.



Illustration drawn in Logo: Courtesy of Ginny Jones, 3/28/2008 retired lecturer, Central Connecticut State University

Fractal Dimensions

Dimensions in Euclidean Geometry

- A point has zero dimension.
- A line (segment) _____ is 1 dimensional.
- A portion of a plane, such as a tabletop or the wall of a classroom, is 2 dimensional.
- A portion of space, such as a box or a classroom is 3 dimensional.

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- Self-similar
- Has the same shape under scaling
- At each point in its construction, the length of the curve increases by a factor of 4/3.
- The resulting figure has infinite length in a finite area of the plane without intersecting itself.
- The curve is more than a line (> 1 D) and yet has no breadth (< 2D).

Fractal Dimension

The conventional Koch curve has a dimension of 1.26 --more than 1 but not 2. Fractal Dimension:

measures the degree of irregularity/ roughness regardless of how much we zoom in on the curve.



Illustration from: *The Science of Fractal Images*; Barnsley, Devaney, Mandelbrot, Peitgen, Saupe and Voss; Springer-Verlag; New York; 1988; page 29. Courtesy of Prof. Dr. Heinz-Otto Peitgen.

Some Relatives of the Koch Curve

- Each is made with
- N = 4 pieces but scaled
- from r = .25 to .5

The dimensions of the shapes change from 1 (a line segment) to 2 (a solid triangle in the plane).



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To play the animation, double click on it. The animation is provided by Prof. Michael Frame from the Mathematics Department at Yale University.

Get QuickTime plug in: <u>http://www.apple.com/quicktime/</u>

Zoom in on the Koch Curve



Zooming in on Koch curve with a dimension approx. =1.26.

To play the animation, double click on it. The animation is provided by Prof. Michael Frame from the Mathematics Department at Yale University.

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Fractal Coastlines





Great Britain



South Africa Fractal dimension = 1*

Fractal dimension = 1.25* Norway

Fractal dimension = 1.52*

Notice that as the fractal dimension increases, the coastline is rougher.

* Dimensions as reported in Eglash, Ron. *African Fractals: Modern Computing and Indigenous Design*. New Brunswick, NJ: Rutgers University Press, 1999. 15. Hand calculations led to the following results: South Africa from Hotagterslip to southeast of Heidelberg: close to 1, Great Britain in the Holyhead region: 1.2 and Norway from south of Namsos to Nesna: 1.5.

Jackson Pollock





- •Jackson Pollock was an action painter.
- •His canvases were large; he laid them on the floor.

•He used the "tool of gesture" of his own human body, to drip (sometimes with sticks), cast (like seeds), spill, spray, splatter, spot, squirt, smear, throw, whip, hurl, and stamp the paint (and other materials at times) on the canvas..

•Hartt. A History of Painting .Sculpture . Architecture. New Jersey / New York: Prentice-Hall, Inc. and Harry N. Abrams, 1985.

Jackson Pollock



Photograph by Hans Namuth: "Pollock Painting, 1950" Autumn Rhythm. Hans Namuth Collection, Center for Creative Photography, The University of Arizona © 1991 ("CCP_JP50_0026_ret.jpg"; archival material) Hans Namuth Estate.



Photograph by Hans Namuth: "Untitled" ca 1950. Hans Namuth Collection, Center for Creative Photography, The University of Arizona © 1991 ("CCP_JP50_021_ret.jpg"; archival material) Hans Namuth Estate.

Jackson Pollock

- Click here to see a movie clip of Jackson Pollock painting: (<u>http://www.nga.gov/feature/pollock/process3qt.shtm</u>)
- Get QuickTime plug in: <u>http://www.apple.com/quicktime/</u>
- View the VHS, Jackson Pollock: Portrait (Strokes of Genius series), by filmmaker, Amanda Pope, for Cort Productions, winner of the Blue Ribbon, American Film Festival; CINE Gold Eagle (ISBN: 1-55974-191-0)



"Composition with Pouring II" (1943) by Jackson Pollock

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Pollock, Jackson. *Composition with Pouring II* 1943, Oil and enamel paint on canvas, 25 x 22 1/8 in. (63.5 cm x 56.2 cm.) Hirshhorn Museum and Sculpture Garden, Smithsonian Institution, Washington DC

Fractal Dimension: close to 1

Fractal Dimensions determined by Taylor, Micolich and Jonas using "Box Counting" method

Reference: Taylor, "Fractal Expressionism", On-line: Internet, available at <u>http://plus.maths.org/issue11/features/physics_world/in</u> dex.html, page 4.



Reference: Taylor, "Fractal Expressionism", On-line: Internet, available at http://plus.maths.org/issue11 /features/physics_world/inde x.html.

"Number 14, 1948" by Jackson Pollock

Fractal Dimension: 1.45

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Pollock, Jackson. *Number 14, 1948: Grey,* 1948: Grey. Oil and enamel paint on canvas, 25 x 22 1/8 in. (63.5 cm x 56.2 cm.) Hirshhorn Museum and Sculpture Garden, Smithsonian Institution, Washington DC

Fractals occur in art too! Here is a painting of Jackson Pollock who sometimes dripped paint on the canvas laid at his feet.



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Pollock, Jackson. *Autumn Rhythm: Number 30*, 1950, Oil on canvas, 8 ft. 10 1/2 in. x 17 ft. 8 in. (270.5 x 538.4 cm.) Collection: Metropolitan Museum of Art, George A. Hearn Fund, 1957.

"Autumn Rhythm: Number 30" (1950)

by Jackson Pollock

Reference: Taylor, "Fractal Expressionism", On-line: Internet, available at http/://materialscience.uoregon.edu/fractal_taylor.html, pp. 4-5. Currently available at <u>http://plus.maths.org/issue11/features/physics_world/index.html</u>. 16

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Pollock, Jackson. *Blue Poles: Number 11*, 1952, Enamel and aluminum paint with glass on canvas, 6 ft. 11 in. x 16 feet (210.8 x 487.6 cm.) Collection: Australian National Gallery, Canberra, Australia

"Blue Poles: Number 11" (1952) by Jackson Pollock Dimension: 1.72

Reference: Taylor, "Fractal Expressionism", On-line: Internet, available at http/://materialscience.uoregon.edu/fractal_taylor.html, page 5. Currently available at http://plus.maths.org/issue11/features/physics_world/index.html.

Activity: Building a Fractal Tetrahedron with Envelopes:

Each student makes 4 pieces and then the activity continues with building additional stages.

Tetrahedron: A solid figure with four sides



Reference: Frame, Michael. Fractal Geometry, Yale University, (Michael Frame, Benoit Mandelbrot, and Nial Neger) *Similarity Dimension*, available at http://classes.yale.edu/fractals/Welcome.html, Date accessed March 30, 2005. Courtesy of Michael Frame.

Four Stages of a Sierpinski Tetrahedron



•The Sierpinski tetrahedron has a fractal dimension of 2.

•Notice how empty a 2dimensional object can be in 3dimensional space!







Reference: Frame, Michael. Fractal Geometry, Yale University, (Michael Frame, Benoit Mandelbrot, and Nial Neger) *Similarity Dimension*, available at <u>http://classes.yale.edu/fractals/Welcome.html</u>, Date accessed March 30, 2005. Courtesy of Michael Frame.