Introduction to Fractal Geometry and its Applications

Workshop 4: Fractals in Nature and Science
The Koch Curve

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Middle School Slides: Fractal Math
Review from Workshop 3:

Prof. Mandelbrot
The Sierpinski Triangle
Fractals

- Measure Roughness
- Broken
- Fragmented
- Irregular

- Concept created by Benoit Mandelbrot

Picture of Benoit B. Mandelbrot was taken at his lecture at Worcester Polytechnic Institute, November 2006 and the picture of the Mandelbrot set is from: The fractal geometry web site, http://classes.yale.edu/fractals/ of Michael Frame, Benoit Mandelbrot and Nial Neger. Courtesy of Michael Frame.
Fill in the stages needed to construct a Sierpinski triangle.
Can you construct the next stage?

See handout.
Fractals in Nature

Lungs: “The lungs, too, need to pack the greatest possible surface into the smallest space. An animal’s ability to absorb oxygen is roughly proportional to the surface area of its lungs. **Typical human lungs pack in a surface bigger than a tennis court.**”


Image of human lung cast courtesy of Prof. Ewald R. Weibel, MD, DSc.
Fractals in Science
Two Moons of Jupiter: Europa and Io

Background Information: Jupiter

Jupiter is a giant gaseous planet. It is the largest planet in the solar system. Jupiter has a swirling “red spot” that seems to be an orderly structure within its turbulent atmosphere. Jupiter’s atmosphere including the red spot can be duplicated in a laboratory by scientists.

The pictures on this page are obtained from NASA’s planetary web site located at http://pds.jpl.nasa.gov/planets/welcome.htm. This site is written on three levels; the children’s level is recommended.

Europa

Europa is a moon of Jupiter about the same size as Earth’s moon. It was first discovered by Galileo in 1610. Scientists are interested in Europa because there could be an ocean under its surface with life forms. Europa has a very icy fractal surface.
**Fractals and Science**

**The Two Moons of Jupiter: Europa and Io**

Io is another large moon of Jupiter; it has more volcanoes than the earth. Io was also discovered by Galileo in 1610. There is a volcanic explosion in the picture at the right at the 11 o’clock position. The Voyager I space probe discovered sulphur dioxide on Io which can freeze very quickly and turn into sulphur dioxide snow. The surface is very **rough and fractal**.
<table>
<thead>
<tr>
<th>Are these fractals?</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broccoli</td>
<td>Yes—Can you find similar pieces on at least 3 levels of magnification?</td>
</tr>
<tr>
<td>Lightning</td>
<td>Yes -- Some of the lightning lines keep splitting in two. This process is called bifurcation and is associated with fractals.</td>
</tr>
<tr>
<td>Bat mosaic from the famous site of the Alhambra in Spain</td>
<td>No – One reason the design is not a fractal is that there is no shrinking or enlarging of the same pattern.</td>
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</table>

Lightning: The fractal geometry Web site, [http://classes.yale.edu/fractals/](http://classes.yale.edu/fractals/) of Michael Frame, Benoit Mandelbrot and Nial Neger is highly recommended for teachers and parents. Illustration courtesy of Michael Frame.
## Comparison of Classical and Fractal Geometry

<table>
<thead>
<tr>
<th>Euclidean Geometry</th>
<th>Fractal Geometry</th>
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</thead>
<tbody>
<tr>
<td>• Ancient: Euclid organized our ordinary geometry in 300 BCE.</td>
<td>• Modern discovery—even though fractals in nature have been around forever</td>
</tr>
<tr>
<td>• Based on size and scale</td>
<td>• Based on shapes being self-similar</td>
</tr>
<tr>
<td>• Suits person-made objects such as tables, football fields and silos</td>
<td>• Describes shapes of nature</td>
</tr>
<tr>
<td>• Usually described by formulas $A = \frac{1}{2}bh, \quad V = l \cdot w \cdot h$, etc.</td>
<td>• Usually based on a building process that gets repeated and repeated</td>
</tr>
</tbody>
</table>

Object | Length
--- | ---
A | 1 ft
B | \( \frac{4}{3} \) ft
C | \( \frac{16}{9} \) ft
D | \( \frac{1}{9} \) ft
E | \( 16 \cdot \frac{1}{9} \) ft

**Koch Curve Directions:**

← (A) Divide a line segment into thirds. Suppose that the original line segment is 1 foot in length.

←(B) Erase the middle portion. Each new segment is 1/3 foot long.

←(C) Over the middle portion, construct two sides of an equilateral triangle. The total length is 1 \( 1/3 \) ft or 4/3 ft since each segment is of a 1/3 foot long and there are 4 segments.

←(D) Now for each of the four line segments, erase their middle portion.

←(E) Again construct the top of an equilateral triangle over the middle portions. Each of the 16 tiny line segments has length of 1/9 ft. The resulting figure is 16 \( 1/9 \) ft long.
Activity: With the **Koch Curve Worksheet**, complete the next stage on your own. Find its length? Hint: What is the length of each of the new line segments? How many are there? Is there a pattern forming here?

Repeating this process over and over gives us the **Koch curve** named after the Swedish mathematician, Neils Fabian Helge von Koch who first introduced the curve in a paper he wrote in 1905. This Koch Curve’s crinkly shape is infinitely long and yet it can be enclosed in a box with a finite area.
In 1905 this seemed very puzzling and people called shapes that acted this way mathematical monsters!

**Activity**: Write a one page paper on the Swedish mathematician, Neils Fabian Helge von Koch (1870-1924).

**Activity**: Try multiplying $4/3$, approximately equal to 1.3, on your calculator by itself over and over again. What happens?
The Koch Curve

The pictures above illustrate an antenna (1) and a portion (2) used in its construction that looks very much like a Koch curve. Historical note: Examine the waves of the famous Japanese painting (3) by Hokusai. Does it appear fractal and similar to the Koch curve? Which part?

Fractal Banisters

• Let’s start with what is called an initiator

• Imagine sliding down a low dimension banister

• Imagine sliding down a high dimension banister

If these two banisters were fractal, which one would you slide down?
The Koch curve has a dimension of 1.26--more than 1 but not 2. **Fractal Dimension:** measures the degree of irregularity or roughness regardless of how much we zoom in on the curve.

Activity: Counting the Upward Triangles of the Sierpinski Triangle

Complete the Sierpinski lab—See the worksheet.

Color your own triangle!