Introduction to Fractal Geometry and its Applications

Workshop 2: Scaling

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Middle School Slides: Fractal Math
Review from Introductory Workshop 1
Fractals

- Measure Roughness
- Fractals can be described as
  - Broken
  - Fragmented
  - Irregular
- Concept created by Benoit Mandelbrot

Benoit B. Mandelbrot

Picture of Benoit B. Mandelbrot was taken at his lecture at Worcester Polytechnic Institute, November 2006 and the picture of the Mandelbrot set is from: The fractal geometry Web site, http://classes.yale.edu/fractals/ of Michael Frame, Benoit Mandelbrot and Nial Neger. Courtesy of Michael Frame.
This is a shoal near the coastline of the Bahamas. It is very jagged and rough.

Reference: Michael Frame, *Natural and Manufactured Fractals*, [http://classes.yale.edu/Fractals/Welcome.html](http://classes.yale.edu/Fractals/Welcome.html).

Courtesy of Michael Frame.
Fractals in Your Body

Fractal networks branch and then branch again; those branches continue to branch and then again and again.

Image of human lung cast courtesy of Prof. Ewald R. Weibel, MD, DSc.

Reference: Michael Frame, *Natural and Manufactured Fractals*, [http://classes.yale.edu/Fractals//welcome.html](http://classes.yale.edu/Fractals//welcome.html).
Fractals occur in art too! Here is a painting of Jackson Pollock who sometimes dripped paint on the canvas laid at his feet.

“Blue Poles: Number 11” (1952) by Jackson Pollock
Dimension: 1.72

Pollock, Jackson. *Blue Poles: Number 11*, 1952, Enamel and aluminum paint with glass on canvas, 6 ft. 11 in. x 16 feet (210.8 x 487.6 cm.) Collection: Australian National Gallery, Canberra, Australia

Geometric Self-similarity
• When two objects have the same shape they are called geometrically similar.
Scaling Factor

- The **scaling factor** compares corresponding parts of similar objects.
- Compare base with base
  \[ \frac{6}{3} = 2 \]
- or height with height
  \[ \frac{8}{4} = 2 \]
- or hypotenuse with hypotenuse
  \[ \frac{10}{5} = 2 \]

Notice that all three ratios are the same and the ratio compares the new part (on the right) with the old part (on the left).
Scaling Factor Problem

- The **scaling factor** compares corresponding parts of similar objects.

- Calculate the scaling factor for the example on the right.

- Notice when enlarging an image, the scaling factor is greater than 1.
Scaling Factor

- The scaling factor compares corresponding parts of similar objects.
- Compute the scaling factor for the example below.

Notice for shrinking an image, the scaling factor is less than 1.
Magnification

Choose a small square in the second diagram and enlarge it to be 4 times wider and 4 times higher. The result is the larger orange square. Now select a smaller square in this diagram.
Magnification

Enlarge the tiny green square to be 8 times wider and 8 times higher. See the result.
Magnification

What is a coastline? Draw an island and point out what is the land and what is the actual coastline.

The first and last slides are the same. As we move across, notice that the small rectangle in each slide is magnified to give the next slide.

Courtesy of Prof. Dr. Heinz-Otto Peitgen for the coastline images from *The Science of Fractal Images*, Barnsley, Devaney, Mandelbrot, Peitgen, Saupe and Voss, Springer-Verlag, New York, 1988, page 23
Activities: Investigate Coastlines

1. With Google maps you can zoom into lots of (although not all) places at a pretty high resolution. When you get to a computer with Web access, connect to [http://maps.google.com/](http://maps.google.com/)

2. Then enter South Africa and zoom into the southern coast using a satellite map and continue zooming in; next type Great Britain and examine the coast south of Dumfries, UK (Be sure to use the satellite map and the down arrows.); and finally examine the northern coastline of Norway. Are they equally rough? Are there other objects of nature that look fractal in the pictures—shoals, rivers, rock formations?
Learning about Fractals and Art

Max Ernst, a German artist, was from an early age enchanted by forests. In Swamp Angel he found fascinating shapes and textures by using the process of "decalcomania." He transferred the designs by pressing the oil paint on the canvas with another surface.

Can you see the angel?

Can you see the fractal and the Euclidean symmetries?

Painting by eighth grade student, Longfellow School, Bridgeport, CT
Bifurcation means splitting in two. Can you see bifurcations in the picture?

Painting by eighth grade student, Longfellow School, Bridgeport, CT
Activity: Change of Pace:  
Learning about Fractals and Art

1. Take small and large sheets of construction paper or heavy white, and somewhat glossy, sheets of paper and thick paint. Latex paint or finger paints are two good choices because they clean up with water.

2. Start with a dab of paint on one small sheet and press a second small sheet on top of it. Quickly pull the sheets apart. Look for many levels of branching. What you are seeing is a fractal structure.

3. Now experiment with your larger paper using 3 dabs of complementary or diverse colors. Press a second sheet on top of it. What do you see when you quickly pull the sheets apart?

4. Take another large sheet and apply the paint in the center. Then carefully fold your sheet in half and repeat the above procedure. What you see are the symmetry of Euclidean geometry and the branching of fractal geometry! How is it like your other sheets and how is it different? Have fun.