

# Fractal Geometry and its Applications

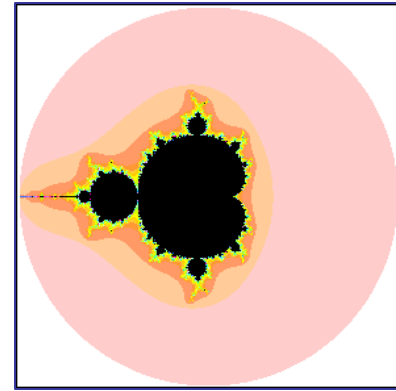
## Workshop 1: Introduction

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Middle School Slides: Fractal Math

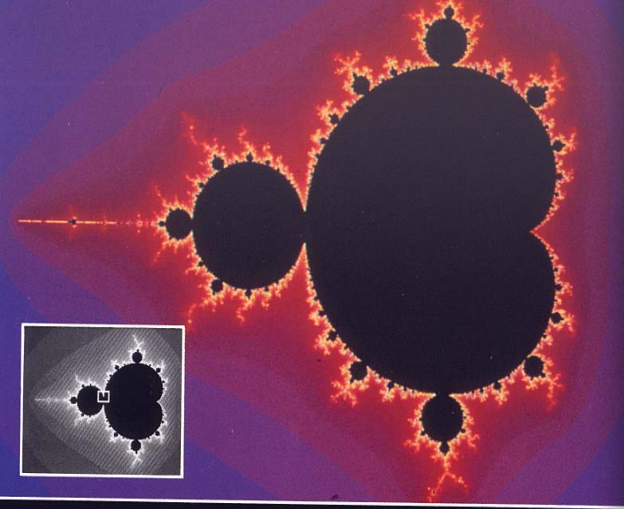
# Fractals

- Fractals were discovered by Benoit Mandelbrot to measure **roughness**
- Fractals can be described as
  - Broken
  - Fragmented
  - Irregular.
- The name, fractal, was created by Prof. Mandelbrot.

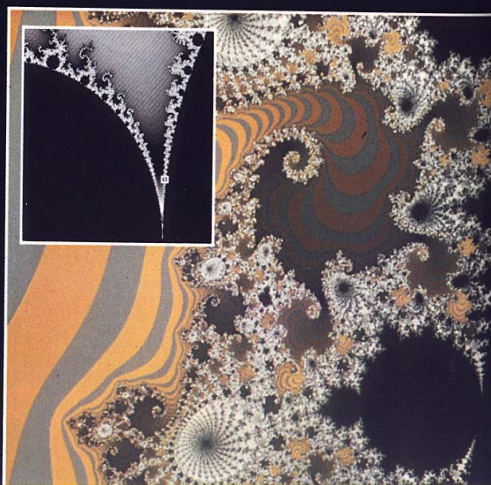


**Benoit B. Mandelbrot**

Picture of Benoit B. Mandelbrot was taken at his lecture at Worcester Polytechnic Institute, November 2006 and the illustration of the Mandelbrot set is from: The fractal geometry Web site, <http://classes.yale.edu/fractals/> of Michael Frame, Benoit Mandelbrot and Nial Neger. Courtesy of Michael Frame.

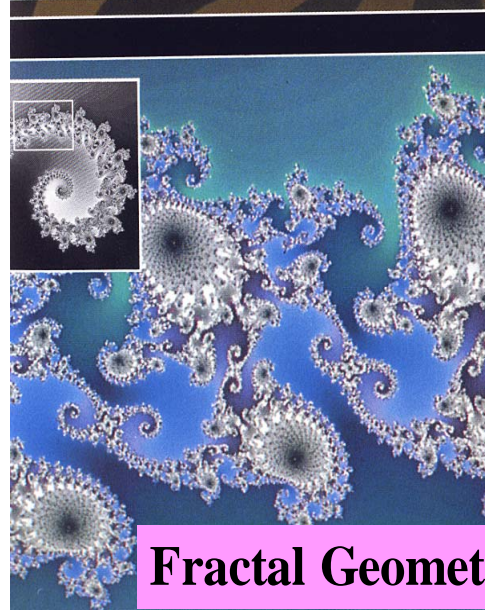
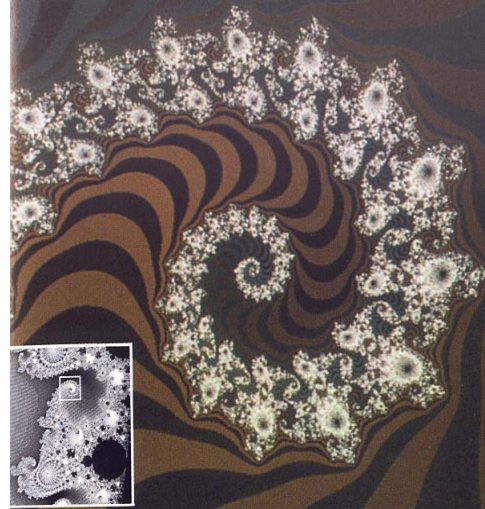


THE MANDELBROT SET. A voyage through finer and finer scales shows the increasing complexity of the set, with its seahorse tails and island molecules resembling the whole set. By the last frame, the level of magnification is about one million in each direction.



Magnifications of the Mandelbrot set courtesy of Prof. Dr. Heinz-Otto Peitgen

From center insert of *Chaos*, James Gleick, Penguin Books, New York, 1987



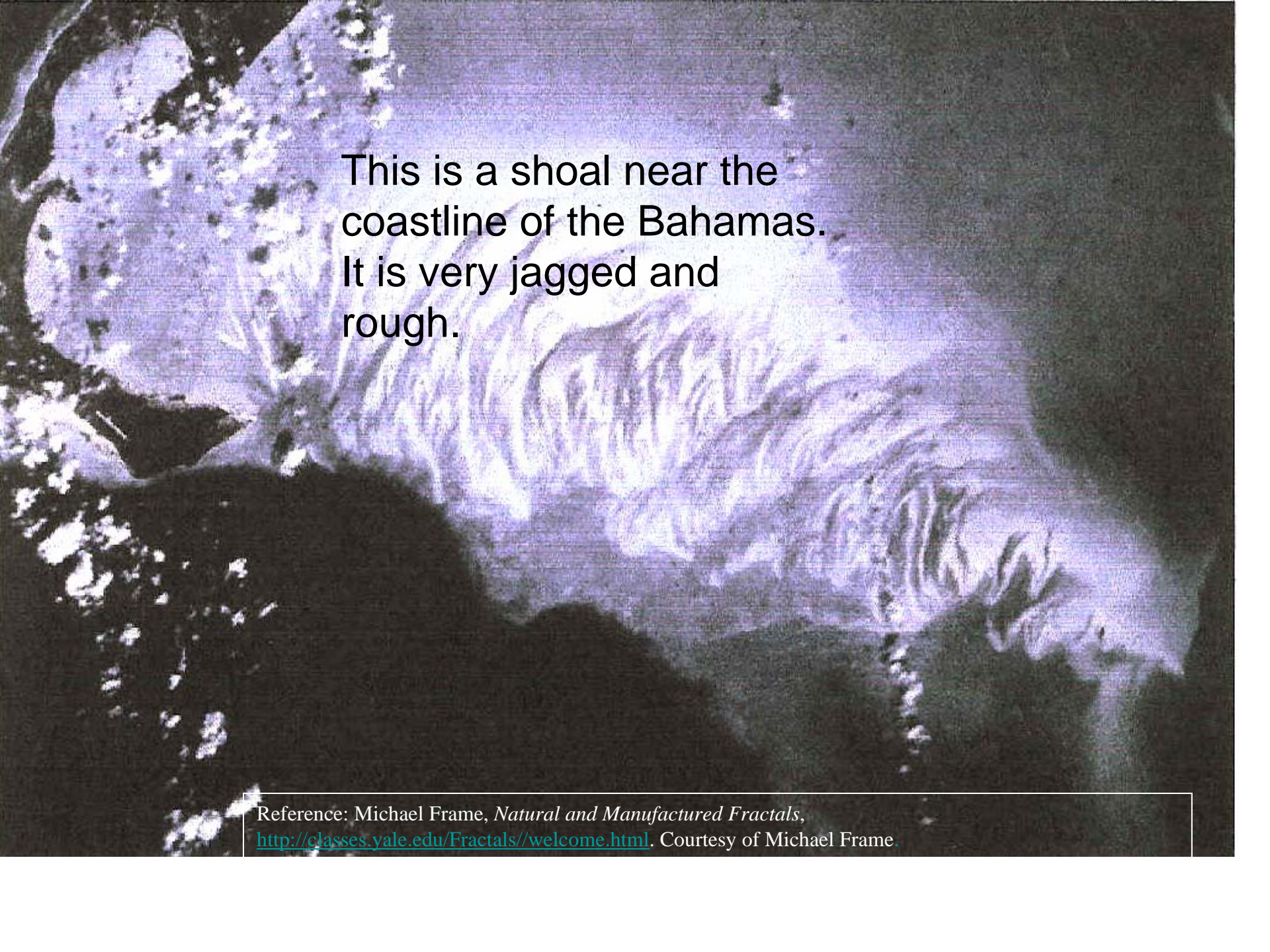
## Fractal Geometry

The Mandelbrot Set

The black bug-like object is called the Mandelbrot set. Sometimes it is called the M set. Mathematicians are investigating its structure. Looking at the zoomed pictures can you see baby Mandelbrot sets and baby baby Mandelbrot sets?

Later, we will see how the Mandelbrot set is governed by a single equation.

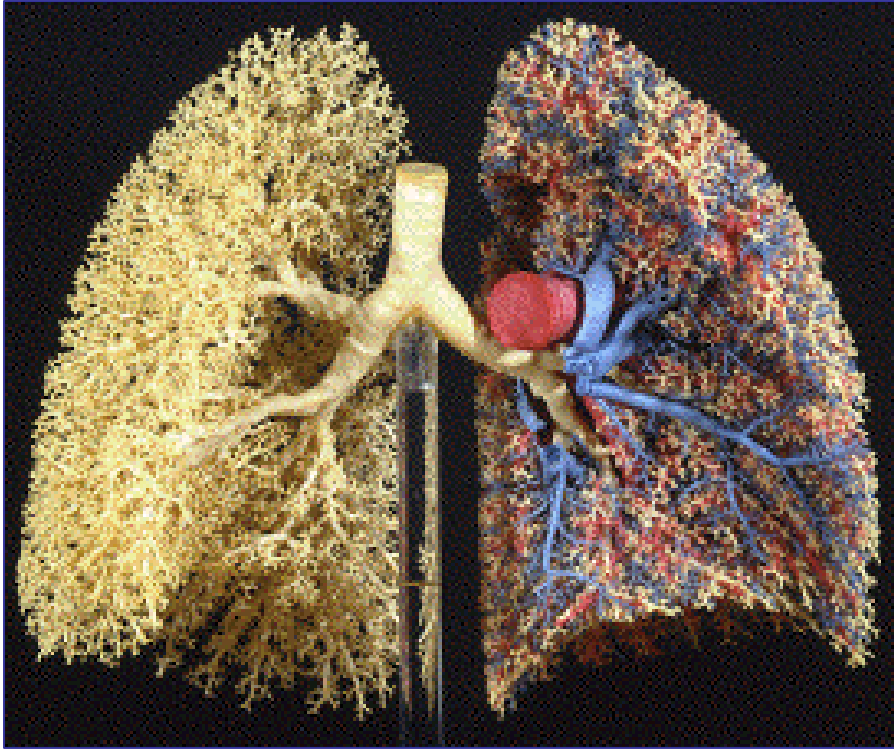


A satellite image of a coastal area, likely near the Bahamas, showing a highly irregular and jagged shoreline. The land is dark, and the surrounding water is light blue. The coastline is characterized by numerous small, intricate details and a complex, non-linear shape, which is a classic example of a fractal in nature.

This is a shoal near the  
coastline of the Bahamas.  
It is very jagged and  
rough.

Reference: Michael Frame, *Natural and Manufactured Fractals*,  
<http://classes.yale.edu/Fractals/welcome.html>. Courtesy of Michael Frame.

# Fractals in Our Human Bodies



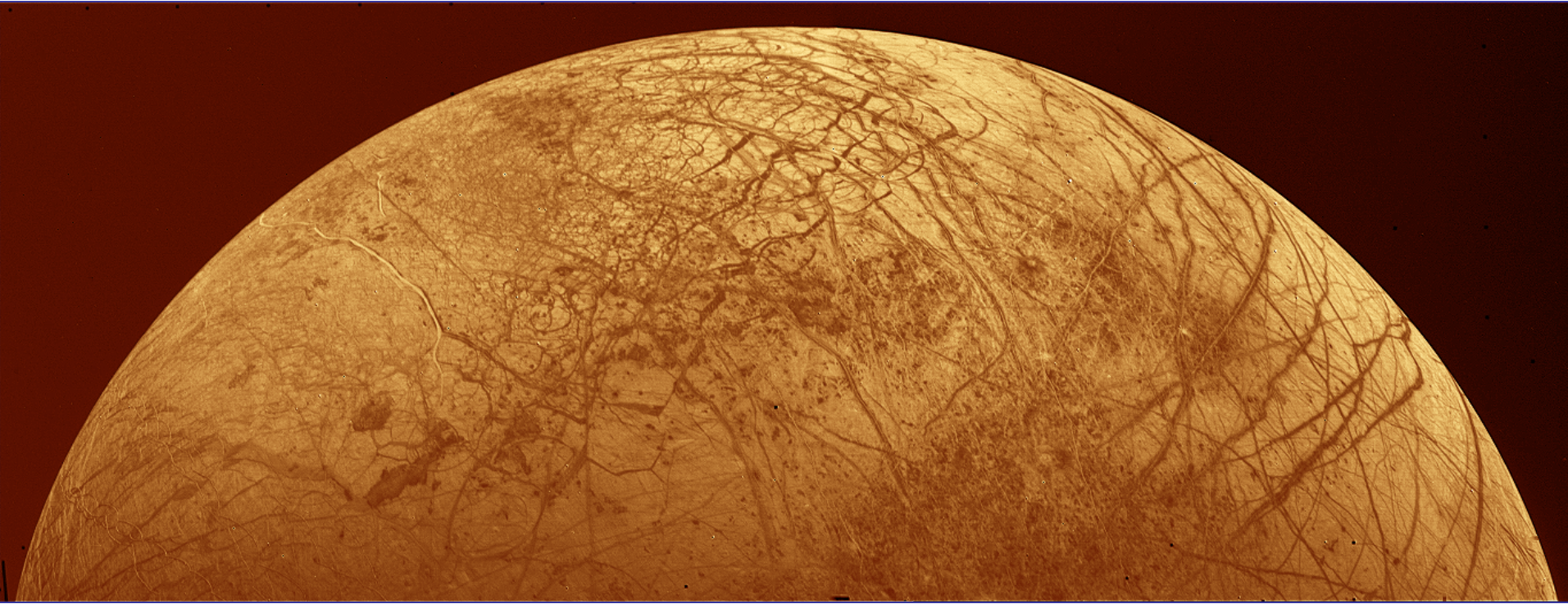
Fractal networks branch and then branch again; those branches continue to branch and then again and again.

Image of human lung cast courtesy of Prof. Ewald R. Weibel, MD, DSc.

Reference: *Natural and Manufactured Fractals* from the fractal geometry Web site, <http://classes.yale.edu/fractals/> of Michael Frame, Benoit Mandelbrot and Nial Neger.



Fractals are important in science. Here the moon of Jupiter called Europa is investigated by NASA.

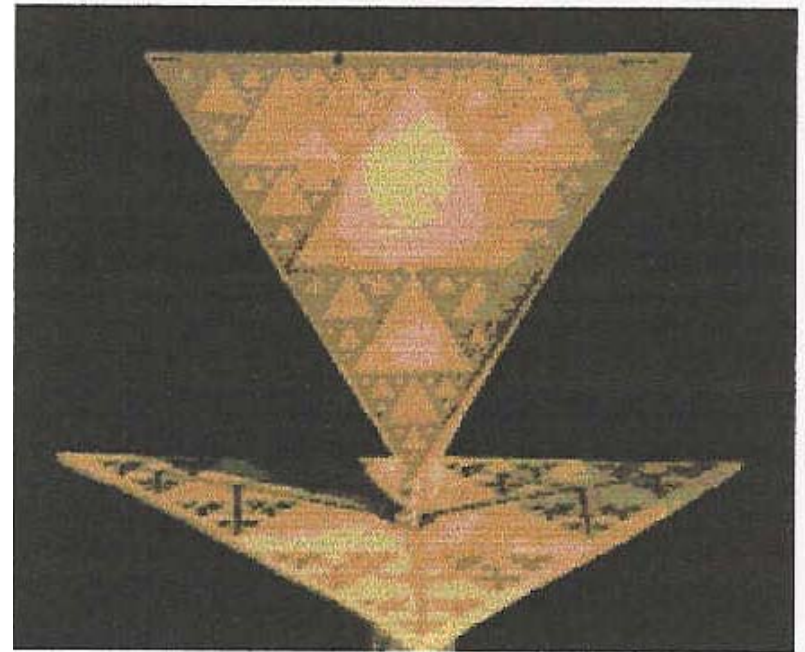
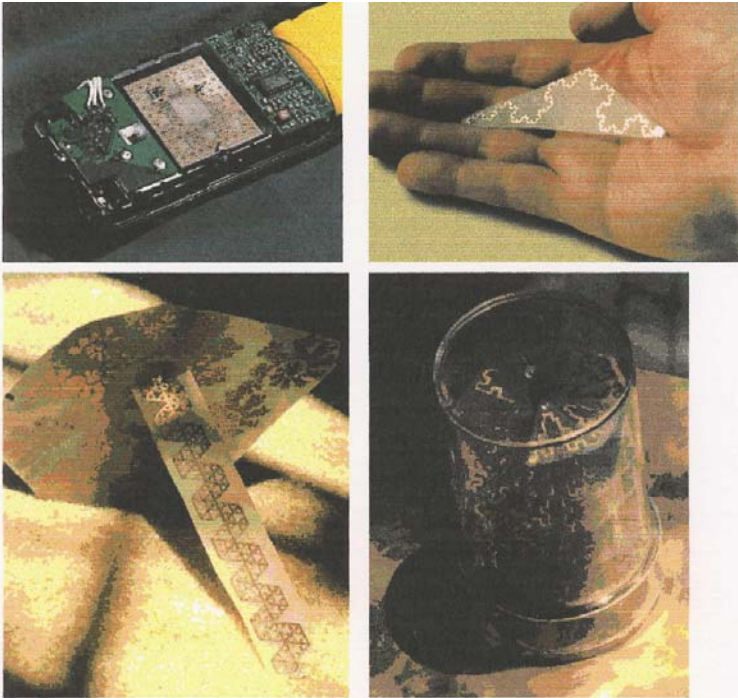


Europa has a very **icy fractal surface**.

NASA's Planetary Web Site,  
<http://pds.jpl.nasa.gov/planets/welcome.htm>,  
Children's Level, Date accessed: October 16,  
2005. Courtesy NASA/JPL-Caltech.

# Fractal Antennas

Fractals are important in building new devices.  
Here is a fractal antenna.



Fractal Antenna' Parts Photo Credit: Nathan Cohen © Fractal Antenna Systems, Inc. Used by Permission.

Reference: *Natural and Manufactured Fractals* from the fractal geometry Web site,  
<http://classes.yale.edu/fractals/> of Michael Frame, Benoit Mandelbrot and Nial Neger.



Fractals occur in art too! Here is a painting of Jackson Pollock who sometimes dripped paint on the canvas laid at his feet.



Courtesy of the Pollock-Krasner Foundation

ARS (Artists Rights Society)

Pollock, Jackson. *Autumn Rhythm: Number 30*, 1950, Oil on canvas, 8 ft.

10 1/2 in. x 17 ft. 8 in. (270.5 x 538.4 cm.) Collection: Metropolitan

Museum of Art, George A. Hearn Fund, 1957.

*Autumn Rhythm*

by Jackson Pollock

Reference: Taylor, "Fractal Expressionism", On-line: Internet, available at [http://materialsscience.uoregon.edu/fractal\\_taylor.html](http://materialsscience.uoregon.edu/fractal_taylor.html), pp. 4-5. Currently available at [http://plus.maths.org/issue11/features/physics\\_world/index.html](http://plus.maths.org/issue11/features/physics_world/index.html).



# How fractal geometry impacts your life today:

- Tools for mathematicians and scientists
- Methods for describing nature and measuring roughness
- Medical procedures based on fractal structures in the human body
- Explorations of planetary objects
- Fractal parts and methods in new technological devices. (The most common application is image compression.)
- Fractals in art

Activity: Do a Goggle search on “image compression” and write a one page report on your findings. If you need some help ask your teacher.

# Additional ideas and activities

- Take apart a fern or cauliflower to see its fractal branching structure. Note how smaller pieces look similar to larger pieces.
- View: “Mandelbrot’s World of Fractals” DVD by Key Curriculum Press (ISBN: 978-1-55953-793-3) and the associated web site: <http://classes.yale.edu/fractals/worldOfFractals.html>
- Visit Prof. Michael Frame’s Yale Web site at <http://classes.yale.edu/fractals/Welcome.html>. Then select 2H: Natural Fractals
- View: *The Colors of Infinity* VHS, (Films for the Humanities FFH 7414-A-NT, 800/257-5126) to learn more about fractals.
- View: *Clouds Are Not Spheres: The Fractal of Benoit Mandelbrot* VHS, (A Gordon’s Film Production, 2001), to learn about the life and work of Benoit Mandelbrot.
- Visit the children’s level of NASA’s Planetary Web site at <http://pds.jpl.nasa.gov/planets/welcome.htm>.
- Visit Prof. Cynthia Lanius’ Web site at Rice University at <http://math.rice.edu/~lanius/frac>.